Aga Khan University Examination Board

Notes from E-Marking Centre on SSC-I Mathematics Examination April/ May 2019

Introduction

This document has been produced for the teachers and candidates of Secondary School Certificate (SSC-I) Mathematics. It contains comments on candidates' responses to the 2019 SSC-I Examination, indicating the quality of the responses and highlighting their relative strengths and weaknesses.

E- Marking Notes

This includes overall comments on candidates' performance on every question and *some* specific examples of candidates' responses which support the mentioned comments. Please note that the descriptive comments represent an overall perception of the better and weaker responses as gathered from the e-marking session. However, the candidates' responses shared in this document represent some specific example(s) of the mentioned comments.

Teachers and candidates should be aware that examiners may ask questions that address the Student Learning Outcomes (SLOs) in a manner that require candidates to respond by integrating knowledge, understanding and application skills they have developed during the course of study. Candidates are advised to read and comprehend each question carefully before writing the response to fulfil the demand of the question.

Candidates need to be aware that the marks allocated to the questions are related to the answer space provided on the examination paper as a guide to the length of the required response. A longer response will not in itself lead to higher marks. Candidates need to be familiar with the command words in the SLOs which contain terms commonly used in examination questions. However, candidates should also be aware that not all questions will start with or contain one of the command words. Words such as 'how', 'why' or 'what' may also be used.

General Observations

Generally it is noted that weaker candidates are not well-versed with the hierarchy of arithmetical, algebraic operations, appropriate formulae and their application. This is generally obstructing their performance in overall paper of General mathematics.

Detailed Comments:

Constructed Response Questions (CRQs)

Question 1:

For two non-empty sets A and B, an **onto function** from A to B is defined as $f_1 = \{(p, 10), (q, 10), (r, 25), (s, 30)\}$.

Using the given information, answer the following parts:

i. Find the domain of f_1.

ii. Find the set A.

iii. Select and write down the possible set B from the given two choices.

Choice I: $\{10, 25, 30\}$

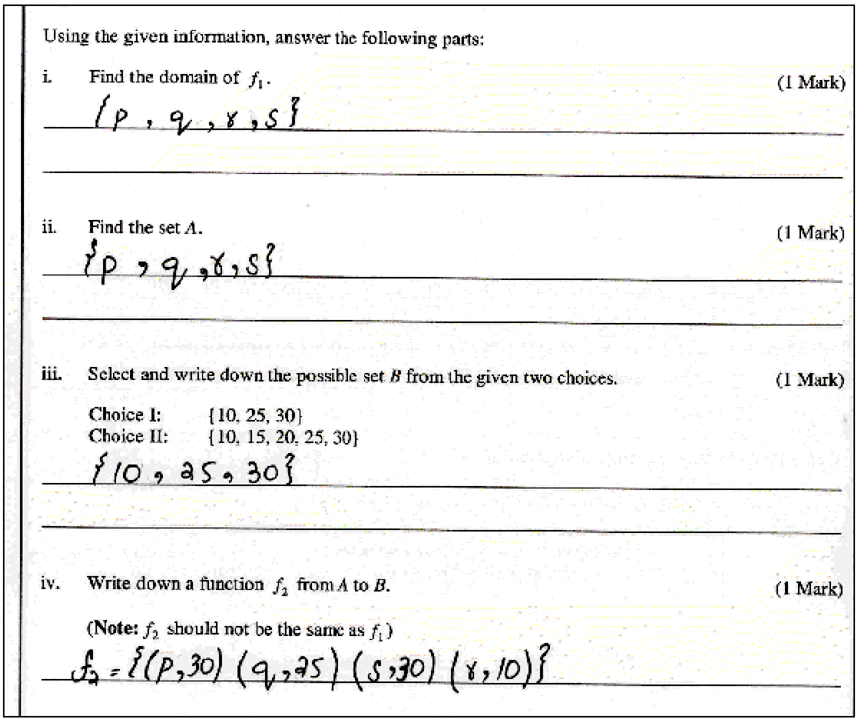
Choice II: $\{10, 15, 20, 25, 30\}$

iv. Write down a function f_2 from A to B.

(**Note:** f_2 should not be the same as f_1)

Better responses exhibited that candidates were able to comprehend the concept of domain, function and onto function and hence were able to found the domain, required set and the function based on the information given in the question.

Example 1:



Using the given information, answer the following parts:

i. Find the domain of f_1. (1 Mark)

$\{p, q, r, s\}$

ii. Find the set A. (1 Mark)

$\{p, q, r, s\}$

iii. Select and write down the possible set B from the given two choices. (1 Mark)

Choice I: $\{10, 25, 30\}$
Choice II: $\{10, 15, 20, 25, 30\}$

$\{10, 25, 30\}$

iv. Write down a function f_2 from A to B. (1 Mark)

(**Note:** f_2 should not be the same as f_1)

$f_2 = \{(p,30)\ (q,25)\ (s,30)\ (r,10)\}$

Weaker responses indicated that candidates have confusions in the concepts of function and its domain and therefore, failed to fulfil the requirement of the question. Some mistakes are cited below.

Example 1:

i. Find the domain of f_1. (1 Mark)

$F_1 = \{(p, 10), (q, 10), (r, 25), (s, 30)\}$.

domain of F_1 = 75.

ii. Find the set A. (1 Mark)

$A = \{10, 10, 25, 30\}$.

set A is equal $\{10, 10, 25, 30\}$

iii. Select and write down the possible set B from the given two choices. (1 Mark)

Choice I: $\{10, 25, 30\}$

Choice II: $\{10, 15, 20, 25, 30\}$

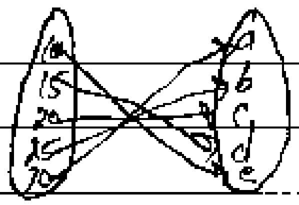
$B = \{10, 15, 20, 25, 30\}$

Set B possible values $\{10, 15, 20\ 25, 30\}$.

iv. Write down a function f_2 from A to B. (1 Mark)

(**Note:** f_2 should not be the same as f_1)

f_2 =



Example 2:

i. Find the domain of f_1. (1 Mark)

$\{10, 25, 30\}$

ii. Find the set A. (1 Mark)

$\{p, q, r, s\}$

iii. Select and write down the possible set B from the given two choices. (1 Mark)

Choice I: $\{10, 25, 30\}$
Choice II: $\{10, 15, 20, 25, 30\}$

~~{10, 25, 30}~~

$\{10, 15, 20, 25, 30\}$

iv. Write down a function f_2 from A to B. (1 Mark)

(Note: f_2 should not be the same as f_1)

~~{(p, 25), (q, 25), (r, 30)}~~

$\{(p, 15), (q, 20), (r, 20), (s, 25)\}$

Question 2:

Simplify $\sqrt[3]{\dfrac{64x^{12}y^{2}}{x^{3}y^{-4}}}$.

Better responses showed a clear understanding of the laws of exponents and its application. They solved the question systematically by appropriate application of laws of exponents to the given situation. They first collected the terms containing the same bases, i.e. x and y and then wrote $\sqrt[3]{4^3x^9y^6}$ as $\left(4^3x^9y^6\right)^{\frac{1}{3}}$ to get final answer.

Example 1:

Solution

$$= \sqrt[3]{\frac{64x^{12}y^{2}}{x^{3}y^{-4}}}$$

$$= \sqrt[3]{64x^{12-3}y^{2+4}}$$

$$= \sqrt[3]{64x^{9}y^{6}}$$

$$= (4^{\cancel{3}})^{1/\cancel{3}}(x^{\cancel{9}^{3}})^{1/\cancel{3}}(y^{\cancel{6}^{2}})^{1/\cancel{3}}$$

$$= 4x^{3}y^{2} \quad \text{Ans}$$

Example 2:

$$\left(\frac{2^{6}x^{12}y^{2}}{x^{3}y^{-4}}\right)^{1/3}$$

$$\left(2^{6}x^{12-3}y^{2+4}\right)^{1/3}$$

$$\left(2^{6}x^{9}y^{6}\right)^{1/3}$$

$$2^{\cancel{6}^{2}\times 1/\cancel{3}_{1}},\ x^{\cancel{9}^{3}\times 1/\cancel{3}_{1}},\ y^{\cancel{6}^{2}\times 1/\cancel{3}_{1}}$$

$$2^{2}x^{3}y^{2}$$

$$\boxed{4x^{3}y^{2}} \quad \text{Ans}$$

Example 3:

$$\sqrt{\frac{2^{6}x^{12}y^{2}}{x^{3}y^{-4}}}$$

$$\left(\frac{2^{6}x^{12}y^{2}}{x^{3}y^{-4}}\right)^{1/3}$$

$$\left(2^{6}x^{12-3}y^{2+4}\right)^{1/3}$$

$$\left(2^{6}x^{9}y^{6}\right)^{1/3}$$

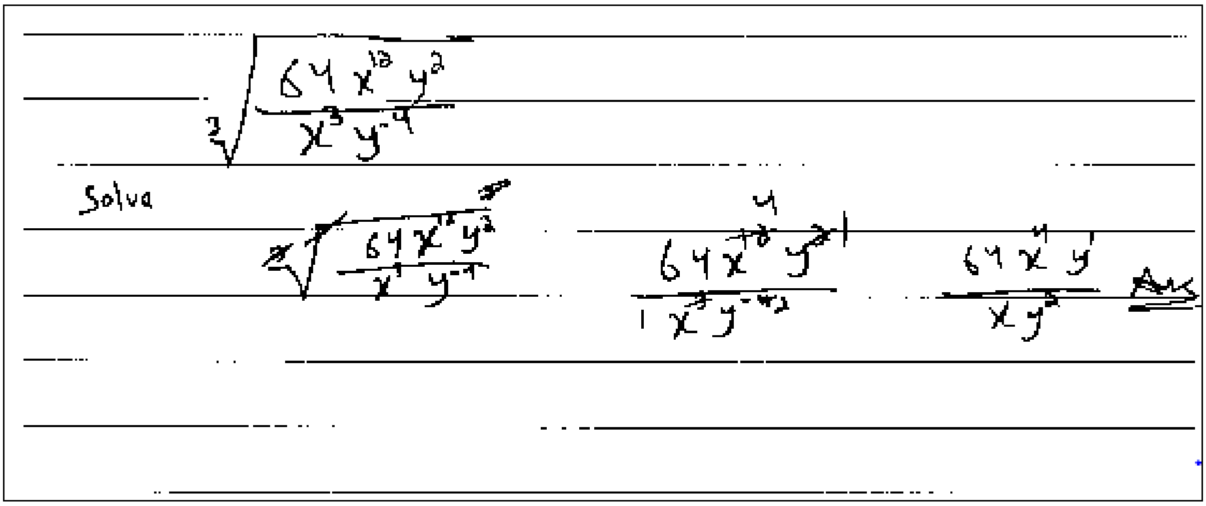
$$2^{\cancel{6}^{2}\times 1/\cancel{3}_{1}},\ x^{\cancel{9}^{3}\times 1/\cancel{3}_{1}},\ y^{\cancel{6}^{2}\times 1/\cancel{3}_{1}}$$

$$2^{2}x^{3}y^{2}$$

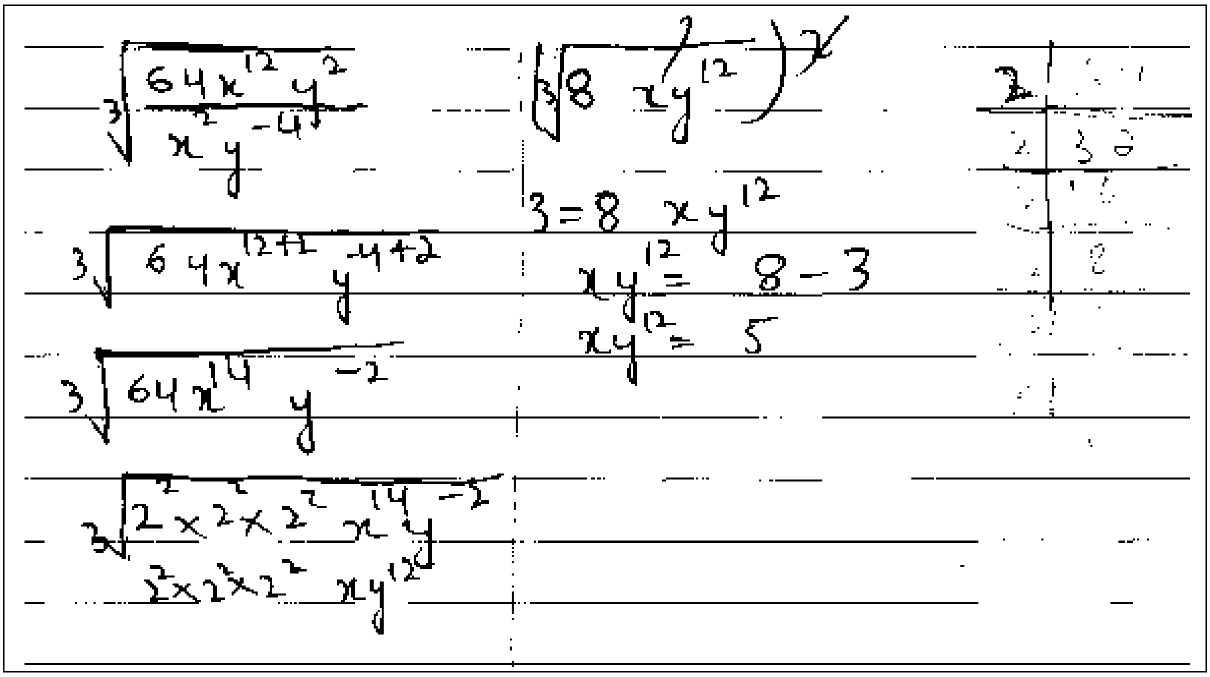
$$\boxed{4x^{3}y^{2}} \quad \text{Ans}$$

Weaker responses showed that the candidates were familiar with the laws of exponents but failed to apply it correctly and made mistakes in the process of simplification. As a result they failed to simplify the given expression. One common mistake is to write $\sqrt[3]{a}$ as $a^{\frac{1}{2}}$. Few other common mistakes noted in the responses are cited in the examples to follow,

Example 1:



Example 2:



Example 3:

$$\sqrt[3]{\frac{64x^{12}y^{2}}{x^{3}y^{-4}}}$$

$$\sqrt[3]{\frac{64x^{12}y^{2}}{x^{3}y^{-4}}} = \sqrt[3]{64x^{12-3}y^{2+4}}$$

$$= \sqrt[3]{64x^{9}y^{6}}^{\frac{1}{3}} \quad \boxed{8x^{9}y^{6}} \text{ Ans.}$$

Question 3:

For the given logarithmic equation $x = \log_3 27 + \log_3 3 - \log_3 3^2$, find the value of *x*.

In *better responses,* candidates applied the laws of logarithm to simplify the given expression to find the value of *x* and able to fulfil the requirement of the question. They correctly wrote $\log_3 3^2 = 2\log_3 3$, $\log_3 3 = 1$ and $\log_3 27$ as $\log_3 3^3$ to find the value of *x* for the given equation.

Example 1:

$$\log_3 \frac{27+3}{3^2} = x$$

$$\log_3 \frac{\cancel{81}^{\cancel{27}\ 9}}{\cancel{9}\cancel{3}_1} = x \Rightarrow \log_3 9 = x$$

$$3^x = 9$$

$$3^x = 3^2$$

$$\boxed{x = 2} \text{ Ans.}$$

Example 2:

$$x = \log_3 27 + \log_3 3 - \log_3 {}^2$$

$$\log_3 27 = x \Rightarrow 3^x = 27 \Rightarrow 3^x = 3^3 \quad x = 3$$

$$\log_3 3 = x \Rightarrow 3^x = 3^1 \Rightarrow x = 1$$

$$\log_3 3^2 = x \Rightarrow 3^x = 3^2 \Rightarrow x = 2$$

$$x = 3 + 1 - 2$$

$$x = 4 - 2$$

$$x = 2$$

Example 3:

$$x = \log_3 27 + \log_3 3 - \log_3 3^2$$

$$x = \log_3 3^3 + \log_3 3 - \log_3 3^2$$

$$x = 3\log_3 3 + \log_3 3 - 2\log_3 3$$

$$\therefore \log_3 3 = 1$$

$$x = 3(1) + 1 - 2(1)$$

$$x = 3 + 1 - 2$$

$$x = 2$$

Weaker responses showed that candidates failed to comprehend the concept of logarithms and its laws. They were unable to apply the laws and failed to simplify $x = \log_3 27 + \log_3 3 - \log_3 3^2$. As a result they were failed to find the value of *x*. Few mistakes observed are cited below.

- $x = \log_3 27 + \log_3 3 - \log_3 3^2 = \log_3 (27 + 3 - 3^2)$
- $x = \dfrac{\log_3 27 \times \log_3 3}{\log_3 9}$
- $x = \dfrac{\log_3 27 + 3}{\log_3 9}$

Example 1:

Solve:- $x = \log_3 27 + \log_3 3 - \log_3 3^2$

$x = \log_3 27 + \log_3 3 - \log_3 9$

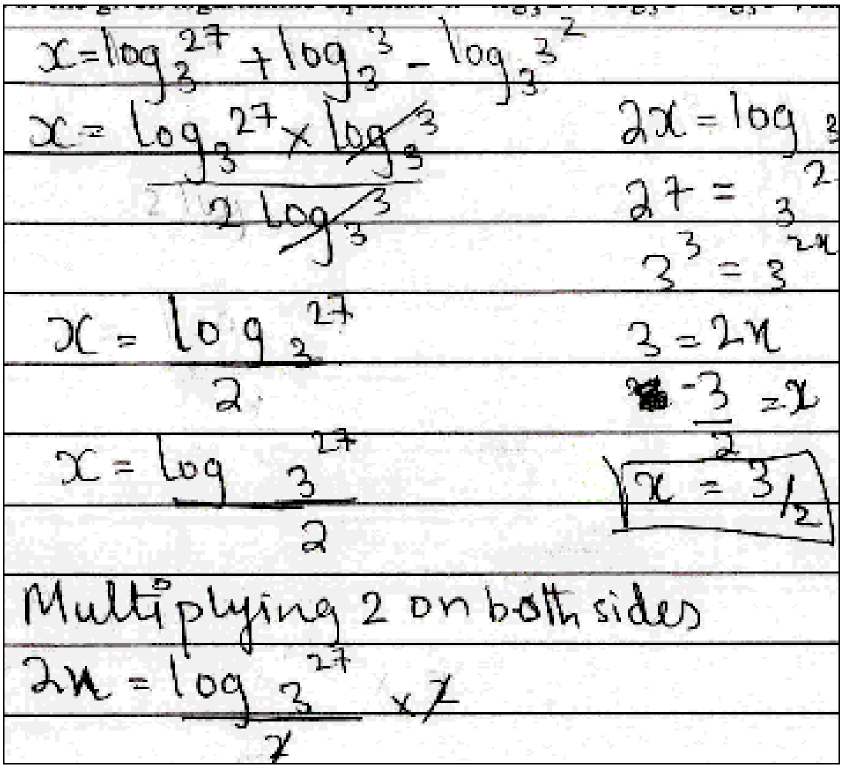
$x = \log_3 9 + \log_3 27 + \log_3 3$

$x = \log_9 39.$

$= \boxed{\log_{39} x = 9}$

Ans

Example 2:



$x = \log_3 27 + \log_3 3 - \log_3 2$

$x = \dfrac{\log_3 27 \times \cancel{\log_3 3}}{2\,\cancel{\log_3 3}}$

$x = \dfrac{\log_3 27}{2}$

$x = \dfrac{\log 3^{27}}{2}$

Multiplying 2 on both sides

$2x = \dfrac{\log 3^{27}}{\cancel{2}} \times \cancel{2}$

$2x = \log_3 27$

$27 = 3^{2x}$

$3^3 = 3^{2x}$

$3 = 2x$

$\dfrac{3}{2} = x$

$\boxed{x = 3/2}$

Question 4a:

For $2x - a = -\dfrac{1}{2x}$ prove that $8x^3 + \dfrac{1}{8x^3} = a^3 - 3a.$

This was generally NOT a well attempted question and candidates were unable to prove the required result.

In *better responses,* candidates took wrote $2x - a = -\dfrac{1}{2x}$ as $2x + \dfrac{1}{2x} = a$, took cube on the both sides, applied the formula and followed the steps systematically to prove the result $8x^3 + \dfrac{1}{8x^3} = a^3 - 3a.$

Example 1:

a) $2n - a = \frac{1}{-2n}$, $2n = -\frac{1}{2n} + a$, $a = 2n + \frac{1}{2n}$

$2n + \frac{1}{2n} = a \Rightarrow a^3 = (2n + \frac{1}{2n})^3$

$a^3 = (2n)^3 + (\frac{1}{2n})^3 + 3(2n)^2(\frac{1}{2n}) + 3(2n)(\frac{1}{2n})^2$

$a^3 = 8n^3 + \frac{1}{8n^3} + 6n + \frac{3}{2n}$

$a^3 = 8n^3 + \frac{1}{8n^3} + 3(2n + \frac{1}{2}h)$

$a^3 = 8n^3 + \frac{1}{8n^3} + 3(a)$ $\therefore a = 2n + \frac{1}{2n}$

$a^3 - 3a = 8n^3 + \frac{1}{8n^3}$ (Proved)

Example 2:

a:- $2x - a = -1/2x$

$2x = -\cancel{1/2x} + a$

$2x + 1/2x = a$

Taking cube on both side

$(2x + \frac{1}{2x})^3 = (a)^3$

$(2x)^3 + \frac{(1)^3}{(2x)^3} + 3(2x)(\frac{1}{2x})(2x + \cancel{1/2x}) = a^3$

$8x^3 + \frac{1}{8x^3} + 3(a) = a^3$

$\boxed{8x^3 + \frac{1}{8x^3} = a^3 - 3a}$

Hence proved

Example 3:

$2x - a = -\frac{1}{2x}$

$2x + \frac{1}{2x} = a$

$2x + \frac{1}{2x} = a$

Cubing both side

$\left(2x + \frac{1}{2x}\right)^3 = (a)^3$ ∵ $(a^3 + b)^3 = a^3 + 3ab(a+b) + b^3$

$(2x)^3 + 3(\cancel{2x})\left(\frac{1}{\cancel{2x}}\right)\left(2x + \frac{1}{2x}\right) + \left(\frac{1}{2x}\right)^3 = a^3$

$8x^3 + \frac{1}{8x^3} + 3(a) = a^3$

$8x^3 + \frac{1}{8x^3} = a^3 - 3a$

Hence, it is proved that

$8x^3 + \frac{1}{8x^3} = a^3 - 3a$ $8x^3 + \frac{1}{8x^3} = a^3 - 3a$

Weaker responses showed that candidates had various misconceptions in proving the required results. At first palce they failed to switch the position of $-\frac{1}{2x}$ and $-a$ in the given equation $2x - a = -\frac{1}{2x}$ and hence tried different formulas and algebraic operations inaptly. The weaker response also showed that candidates have confusion in selection of the right formula between $(a+b)^3$ and $a^3 + b^3$.

Example 1:

Part a

$a^3+b^3=(a+b)(a^2-ab+b^2)$

$8x^3+\frac{1}{8x^3}=\left(2x+\frac{1}{2x}\right)\left((2x)^2-(2x)\left(\frac{1}{2x}\right)+\left(\frac{1}{2x}\right)^2\right)$

$8x^3+\frac{1}{8x^3}=\left(2x+\frac{1}{2x}\right)\left(4x^2-(1)+\frac{1}{4x^2}\right)$

$=4(4$

$8x^3+\frac{1}{8x^3}=\left(2x+\frac{1}{2x}\right)\left(2x-\frac{1}{2x}\right)$

$8x^3+\frac{1}{8x^3}=\left(2x^3+\frac{1}{2x^3}\right)\left(2x^3-\frac{1}{2x^3}\right)$

$8x+\frac{1}{8x}=\left(2x^3-\frac{1}{2x^3}\right)$ Answer

Example 2:

$8x^3+\left(\frac{1}{8x}\right)^3$ [illegible]

$2x+a=\frac{-1}{2x}$

qubing both side

$(2x+a)^3=\left(\frac{-1}{2x}\right)^3$

$(2x)^3+3(2x)^2(a)+3(2x)(a^2)+(a)^3=\frac{1^3}{2x^3}$

$8x^3+6x^2a+6xa^2+a^3=\frac{1}{8x^3}$

$8x^3+xa+a^3=\frac{1}{8x^3}$

$8x^3-\frac{1}{8x^3}=-xa+a^3$

Example 3:

$8x^3 + \frac{1}{8x^3}$ = [illegible]

$2x + a = -\frac{1}{2x}$

quabing both side

$(2x+a)^3 = \left(\frac{1}{2x}\right)^3$

$(2x)^3 + 3(2x)^2(a) + 3(2x)(a^2)+(a)^3 = \frac{1^3}{2x^3}$

$8x^3 + 6x^2a + 6xa^2 + a^3 = \frac{1}{8x^3}$

$8x^3 + xa + a^3 = \frac{1}{8x^3}$

$8x^3 - \frac{1}{8x^3} = -xa + a^3$

Question 4b:

Prove that $(2x+3y)(2x-3y)(4x^2+6xy+9y^2)(4x^2-6xy+9y^2) = 64x^6 - 729y^6$.

Better responses exhibited that candidates correctly proved the result. They used the different approaches to prove the result like simple multiplication and continued multiplication by using formulae of a^3-b^3, a^3+b^3 and followed by formula of a^2-b^2. The candidates which used simple multiplication really had a long process which consists of several steps.

Example 1:

(B) $L \cdot H \cdot S = (2x+3y)(2x-3y)(4x^2+6xy+9y^2)(4x^2-6xy+9y^2)$

$(2x+3y)(4x^2-6xy+9y^2)(2x-3y)(4x^2+6xy+9y^2)$

$\{(2x)^3+(3y)^3\} \times \{(2x)^3-(3y)^3\}$

$\{(2x)^3+(3y)^3\} \times \{(2x)^3-(3y)^3\}$

$\{8x^3+27y^3\} \times \{8x^3-27y^3\}$

$= 64x^6-729y^6 = 64x^6-729y^6$

$L \cdot H \cdot S = R \cdot H \cdot S$

hence proved!

Example 2:

b. $(a+b)(a-b) = a^2 - b^2$ $(L \cdot H \cdot S)$.

$(2x)^2-(3y)^2(4x^2+6xy+9y^2)(4x^2-6xy+9y^2)$

$(4x^2-9y^2)(16x^4 - \cancel{24x^3y} + 36x^2y^2 + \cancel{24x^3y} - \cancel{36x^2y^2} + \cancel{36xy^3} + 36x^2y^2$

$- \cancel{36xy^3} + 81y^4)$

$(4x^2-9y^2)(16x^4+36x^2y^2+81y^4)$.

$(R \cdot H \cdot S)$.

$64x^6-729y^6$

$\therefore a^3-b^3 = (a-b)(a^2+ab+b^2)$

$= (4x^2-9y^2)((4x^2)^2+(4x^2)(9y^2)+(9y^2)^2)$

$= (4x^2-9y^2)(16x^4+36x^2y^2+81y^4)$.

$L \cdot H \cdot S = R \cdot H \cdot S$.

hence proved.

Weaker responses displayed different misconceptions in proving the required result. These misconceptions include wrong application of formulae, wrong provision of signs in the process of multiplication and incapability of arithmetic operations on algebraic terms. Few misconceptions are cited in the following examples.

Example 1:

$(2x-3y)$

$(2x+3y)(4x^2+6xy+9y^2)(2x-3y)(4x^2-6xy+9y^2)=64x^6-729y^6$

$(a^3+b^3)=a^3-b^3 = 64x^6-729y^6$

$a^2-b^2 = 64x^6-729y^6$

$(a^3)^2-(b^3)^2 \quad 64x^6-729y^6$

$(4x^2)^3-(9y^2)^3 \quad 64x^6-729y^6$

$64x^6-729y^6 = 64x^6-729y^6$ Ans.

Example 2:

$(2x+3y)(2x-3y)(4x^2+6xy+9y^2)(4x^2-6xy+9y^2)=64x^6-729y^6$

$(2x^2)(2x^2)+(4x^2)+(4x^2)+(6xy)-(6xy)+(3y)-(3y)(9y^2)(9y^2)=64x^6-729y^6$

$(4x^8)+(16x^4)-(9y)^2+(81y^4)=64x^6-729y^6$

$64x^6-729y^6=64x^6-729y^6$

Example 3:

$$(2x+3y)(2x-3y)(4x^2+6xy+9y^2)(4x^2-6xy+9y^2) = 64x^6 - 729y^6$$

$$= (2x-3y)^2\,(4x+9y)^4$$

$$= (2x^2 \times 4x^4)\,(-3y^2 \times 9y^4)$$

$$= (8x^4)^2 - (27y^4)^2$$

$$= (64x^6) - (729y^6)$$

$$= 64x^6 - 729y^6$$ ∴ Hence its proven that

$$(2x+3y)(2x-3y)(4x^2+6xy+9y^2)(4x^2-6xy+9y^2) = 64x^6 - 729y^6$$

Question 5a:

Factorise the expression $15x^3 - 24x^2 + 3x + 6x$.

The question offered choice a between part **a** and part **b**. Most of the candidates attempted part **a.**

Better responses indicated that candidates had command over the process of quadratic of factorisation. They first added the terms $3x$ and $6x$, took $3x$ common from the given algebraic expression and applied the method of breaking of middle term. Finally, they took the common to complete the factorisation of the algebraic expression.

Example 1:

$15x^3 - 24x^2 + 3x + 6x$

$3x(5x^2 - 8x + 1 + 2)$

$3x(5x^2 - 8x + 3)$

$3x(5x^2 - 3x - 5x + 3)$

$3x(x(5x-3) - 1(5x-3))$

$\cancel{3x(x+1)}$ $3(x-1)(5x-3)$

Example 2:

(a) $15x^3 - 24x^2 + 3x + 6x$

$= 15x^3 - 24x^2 + 9x$

Taking $3x$ common

$= 3x(5x^2 - 8x + 3)$

$= 3x(5x^2 - 3x - 5x + 3)$

$= 3x\{x(5x-3) - 1(5x-3)\}$

$= 3x[(x-1)(5x-3)]$

$= 3x(x-1)(5x-3)$ Ans

Weaker responses indicated that candidates were failed to add the terms $3x$ and $6x$ tried to rearrange the terms, which was not the appropriate way to factorise the given algebraic expression. It is also noted that candidates applied cubic formula or tried completing square method which is a clear indication of a misconception. Incorrect use of algebraic operations was evident in many responses.

Example 1:

4) $15x^3 - 24x^2 + 3x + 6x$

$15x^3 - 6x + 3x + 24x^2$

$15x^3 - 6x + 27x^3$

$15x^3 + 27x^3 - 6x$ Ans

Example 2:

$= 15x^3 - 24x^2 + 3x + 6x$

$= 3x(15x^2 - 24x + 3 + 6)$

$= 3x(5x^2 - 8x + 1 + 2)$

$= \frac{1}{3x} \rightarrow \underline{\underline{Ans}}$

Example 3:

a. $15x^3 - 24x^2 + 3x + 6x$

$\cancel{a^2 + 2ab + b^2}$

$(a+b)(a^2 + ab + b^2)$

$(15x + 3x)((15x)^2 + (15x)(3) + (3)^2$

$18x = 15x + (15x)(3) + 3^2$

$18x = 15x + 45 + 3$

$18x = 63x$

$x = \frac{18}{63^2} \Rightarrow Ans$

Question 5b:

Using factorisation, show that the expression $64a^4 + b^4$ can be written as

$(8a^2 + b^2 + 4ab)(8a^2 + b^2 - 4ab)$.

Better responses indicated that the candidates were able to factorise the given expression $64a^4 + b^4$ to prove the required result. They added and subtracted the correct term to make the given expression a perfect square and aptly applied the formula of $(a)^2 - (b)^2$ and proved that $64a^4 + b^4 = (8a^2 + b^2 + 4ab)(8a^2 + b^2 - 4ab)$.

Example 1:

$64a^4 + b^4$

As $a^2 + 2ab + b^2 = (a+b)^2$

$= (8a^2)^2 + 2(8a^2)(b^2) + (b^2)^2 - 16a^2b^2$

$= (8a^2 + b^2)^2 - 16a^2b^2$

$= (8a^2 + b^2)^2 - (4ab)^2$

$=$ As $(a)^2 - (b)^2 = (a+b)(a-b)$

$= (8a^2 + b^2 + 4ab)(8a^2 + b^2 - 4ab)$

Hence proved

Example 2:

Part B

$\cancel{64a^4+b^4} \quad 64a^4+b^4$

as $a^2+2ab+b^2=(a+b)^2$

$(8a^2)^2+2(8a^2)(b^2)+(b^2)^2-16a^2b^2$

$(a+b)^2$

$(8a^2+b^2)^2-16a^2b^2$

8

$(8a^2+b^2)^2-(4ab)^2$

as $a^2-b^2=(a+b)(a-b)$

$\left((8a^2+b^2)+(4ab)\right)\left(8a^2+b^2-(4ab)\right)$

$(8a^2+b^2+4ab)(8a^2+b^2-4ab)$

Hence it can also written as

$64a^4+b^4=(8a^2+b^2+4ab)(8a^2+b^2-4ab)$

Weaker responses indicated that candidates failed to comprehend the question. Although it was clearly mentioned in the question to use factorisation to express $64a^4+b^4$ as $(8a^2+b^2+4ab)(8a^2+b^2-4ab)$ but candidate used multiplication method. Other weaker response reflected that candidates had idea of adding and subtracting a term in the expression to make it a complete square, but failed to exhibit it properly and subsequently unable to mee the requirement of the question. They also made mistakes as mentioned in the following examples:

Example 1:

$64a^4+b^4 = (a+b)^2 \quad 64$

$= (8a^2+b^2)\ (8a^2-b^2 \quad 8 \quad 8$

$a^2+2ab+b^2 \qquad a-2ab+b^2.$

$(3a)^2+2(2a)(b)+b^2 = (3a)^2-2(2a)(b)+(b)^2.$

$= 8a^2+b^2+4ab \quad = 8a^2+b^2-4ab.$

Example 2:

$(64a^4 + b^4)$

$64a^4 + b^4$

$(64a^2)^2 + (b^2)^2$

$(8a^2 + b^2)^2$

$(8a^2 + 2(2a)(b) + b^2)^2 - 4ab$

$(8a^2 + b^2 + 4ab)(8a^2 - b^2 - 4ab)$

Question 6:

If y is directly proportional to $\sqrt{x}$, then find the missing values of x and y in the following table.

x	16	3	**?**
y	4	**?**	9

This question was based on the concept of variation.

Better responses showed that candidates had good understanding of the concept of variation and were able to write mathematical expression for the given situation in the question. They correctly found the value of constant of proportionality and missing vales of x and y.

Example 1:

$y \propto \sqrt{x}$ (Given)	If $x = 3$ then $y = ?$
$y = k\sqrt{x}$	From the equation, $\sqrt{x} = \frac{y}{k}$ and $y = k\sqrt{x}$
$k = \frac{y}{\sqrt{x}}$	$y = (1)(\sqrt{3})$
$k = \frac{4}{\sqrt{16}}$	$y = \sqrt{3}$ or $3^{1/2}$
$k = \frac{4}{4}$	If $y = 9$ then $x = ?$
$k = 1$	$\sqrt{x} = \frac{y}{k}$
	$\sqrt{x} = \frac{9}{1}$
	$(\sqrt{x})^2 = (9)^2$ (square on both side)
	$x = 81$

Example 2:

$y \propto \sqrt{x}$	$y = k\sqrt{x}$	$y = k\sqrt{x}$
$y = k\sqrt{x}$	$y = 1(\sqrt{3})$	$9 = 1\sqrt{x}$
$4 = k\sqrt{16}$	$y = \sqrt{3}$	$9 = \sqrt{x}$
$4 = k\,4$		Square on both side
$4/4 = k$		$(9)^2 = (\sqrt{x})^2$
$k = 1$		$81 = x$
Verificatication		
① $y = k\sqrt{x}$	② $y = k(\sqrt{3})$	$y = k\sqrt{x}$
$4 = 1\sqrt{16}$	$\sqrt{3} = 1\sqrt{3}$	$9 = 1\sqrt{81}$
$4 = 4$	$\sqrt{3} = \sqrt{3}$	$9 = 1(9)$
		$9 = 9.$
	Proved	

Weaker responses exhibited lack of understanding of the concepts of variation and made different mistakes. One common mistake noted was as follows:

$y = k \propto \sqrt{x}$ but wrote $y \times \sqrt{x} = k$ as a result they found wrong value of k, x, and y.

The following examples depict few more mistakes observed in the weaker responses.

Example 1:

$x = ?$ $\quad x = 6$

$y = ?$

I mean $x = 6$ and $y = 36$.

$y = \frac{9}{y}$

$y = y \times 9$

$y = 36$

$x = \sqrt{y}$

$x = \sqrt{36}$

Example 2:

$y \propto \sqrt{x}$ 3rd Condition

$y = k\sqrt{x}$ $y \propto \sqrt{x}$

$4 = k\sqrt{16}$ $9 = k\sqrt{x}$

$4 = k\sqrt{16}$. Taking Square root $9 = k\sqrt{3}$

$= 4 \times 4$ $= 9 \times 3$

st condition $= 16$ $x = 27$

$x = 16$

2n condition

$y \propto \sqrt{x}$

$y = k\sqrt{x}$ $\longrightarrow$ eq

$3 = k\sqrt{x}$

$3 = k\sqrt{16}$

$. = 3 \times 4$

$y = 12$

Example 3:

If $y \propto \sqrt{u}$

then $y = k\sqrt{u}$

where k is constant ... (i)

putting the values in the first eq (i)

$y = k\sqrt{u}$

$4 = k\sqrt{6}$

$4 = k16$

$16/4 = k$

$4 = k$ putting the value in eq (i)

$y = 16(4)$

$y = 64$ putting the value

* $y = 4u$ $u = 16$ Answer.

* $64 = 4u$

$64/4 = u$

Question 7a:

For the given matrix $A = \begin{bmatrix} 2 & 3 \\ 6 & 2 \end{bmatrix}$, find its

i. determinant.
ii. adjoint.
iii. inverse.

Question 7b:

Using result of part a, show that $A^{-1} \times A = I$.

This question was based on the concept of determinant, adjoint and inverse of the matrices

Better responses informed that candidates found the determinant, adjoint and inverse of the given matrix systematically and followed all the necessary steps.

Better responses displayed that candidates were able to accomplish the requirement of the part a. Thus it was just correct multiplication of two matrices and they did it aptly.

Example 1:

$\Rightarrow |A| = (2)(2) - (6)(3)$

$\Rightarrow |A| = 4 - 18$

$\Rightarrow |A| = -14$

ii. adjoint. (1 Ma

Adjoint of $A = \begin{bmatrix} 2 & -3 \\ -6 & 2 \end{bmatrix}$

iii. inverse. (1 Ma

$\Rightarrow A^{-1} = \frac{1}{|A|} \times Adj.A$

$\Rightarrow A^{-1} = \frac{1}{-14} \times \begin{bmatrix} 2 & -3 \\ -6 & 2 \end{bmatrix}$

$\Rightarrow A^{-1} = \begin{bmatrix} 2/-14 & -3/-14 \\ -6/-14 & 2/-14 \end{bmatrix}$

$\Rightarrow A^{-1} = \begin{bmatrix} -1/7 & 3/14 \\ 3/7 & -1/7 \end{bmatrix}$

b. Using result of part a, show that $A^{-1} \times A = I$. (2 Mar

$A^{-1} \times A = \begin{bmatrix} -1/7 & 3/14 \\ 3/7 & -1/7 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 6 & 2 \end{bmatrix}$

$A^{-1} \times A = \begin{bmatrix} (-1/7)(2) + (3/14)(6) & (-1/7)(3) + (3/14)(2) \\ (3/7)(2) + (-1/7)(6) & (3/7)(3) + (-1/7)(2) \end{bmatrix}$

$A^{-1} \times A = \begin{bmatrix} (-2/7) + (9/7) & (-3/7) + (3/7) \\ (6/7) + (-6/7) & (9/7) + (-2/7) \end{bmatrix}$

$A^{-1} \times A = \begin{bmatrix} 7/7 & 0 \\ 0 & 7/7 \end{bmatrix}$

$A^{-1} \times A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Weaker responses showed that candidates made following mistakes:

- Wrong calculation of determinant of B
- Failed to calculate adjoint of given matrix and hence were unable to calculate inverse of the given correctly
- They made mistakes in the signs while finding determinant, adjoint and inverse of given matrix.
- The common mistake observed in the matrix multiplication is $A^{-1} \times A = \begin{bmatrix} 2 & -3 \\ -6 & 2 \end{bmatrix} \times \begin{bmatrix} 2 & 3 \\ 6 & 2 \end{bmatrix}$ is $\begin{bmatrix} 2\times 2 & -3\times 3 \\ -6\times 6 & 2\times 2 \end{bmatrix}$

  Few other mistakes are cited in the following examples

Example 1:

i. determinant.

$$\begin{bmatrix} -2 & 3 \\ 6 & -2 \end{bmatrix}$$

ii. adjoint.

$$A = \begin{bmatrix} 2 & 3 \\ 6 & 2 \end{bmatrix} = (2)(2) - (3)(6) = 4 - 18$$
$$= -14$$

iii. inverse.

$$\begin{bmatrix} -3 & 2 \\ 2 & -6 \end{bmatrix}$$

b. Using result of part a. show that $A^{-1} \times A = I$.

$$\begin{bmatrix} -2 & 3 \\ 6 & -2 \end{bmatrix} x \begin{bmatrix} 2 & 3 \\ 6 & 2 \end{bmatrix}$$

Example 2:

i. determinant. (1 M

$$\det A = \begin{bmatrix} 2 & -3 \\ -6 & 2 \end{bmatrix}$$

ii. adjoint. (1 M

adj $A = [18 \quad 4]$ or $[72]$

iii. inverse. (1 M

$$A^{-1} = \begin{bmatrix} 144 & -216 \\ -432 & 144 \end{bmatrix}$$

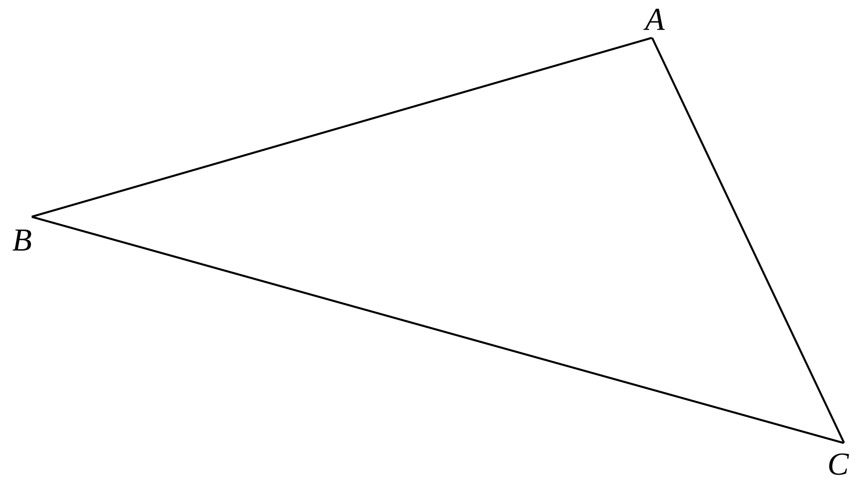
b. Using result of part a, show that $A^{-1} \times A = I$. (2 Ma

$$I = \begin{bmatrix} 144 & -216 \\ -432 & 144 \end{bmatrix} \times \begin{bmatrix} 2 & 3 \\ 6 & 2 \end{bmatrix} \qquad I = \begin{bmatrix} 288 & -648 \\ -2592 & 288 \end{bmatrix}$$

$$I = \begin{bmatrix} (144)\times(2) & (-216)\times(3) \\ (-432)\times(6) & (144)\times(2) \end{bmatrix}$$

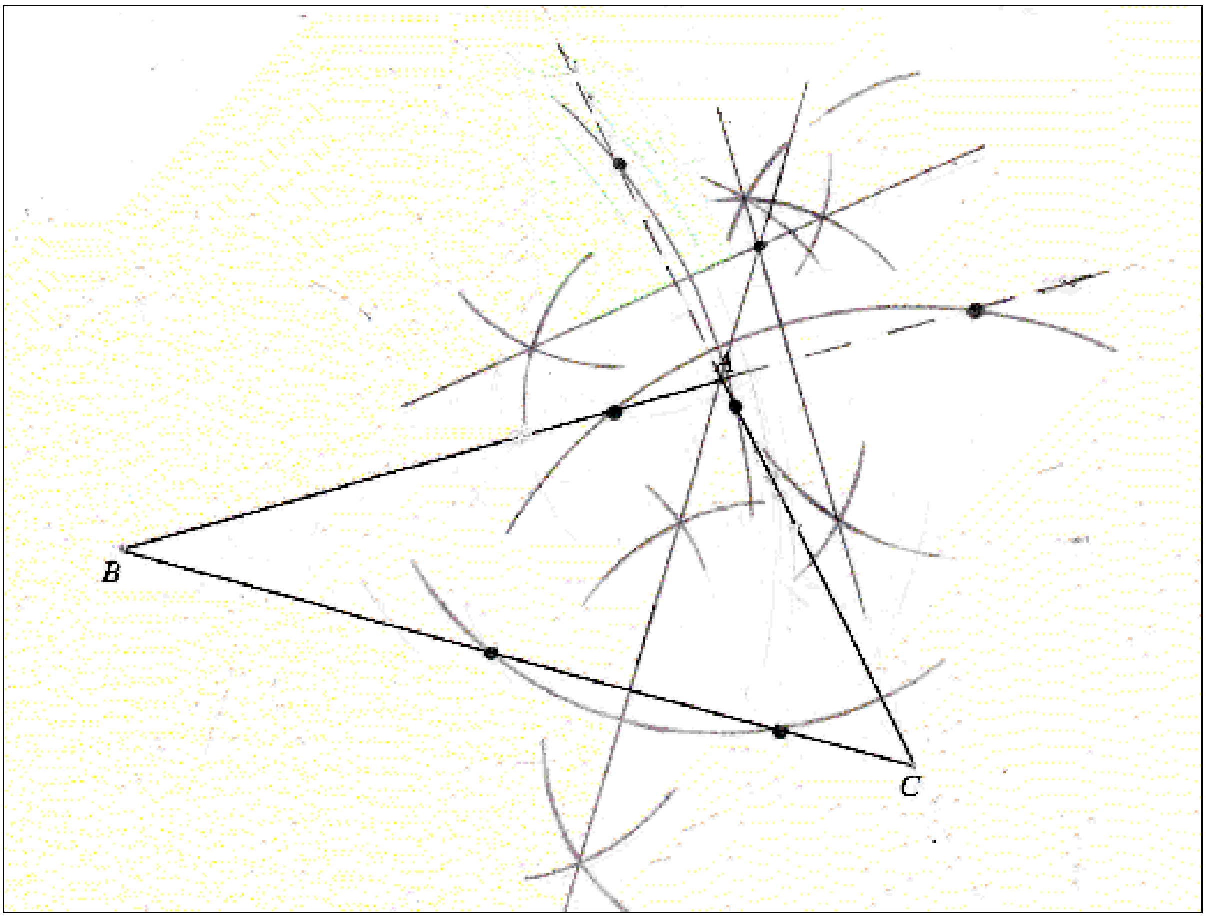
Question 8:

For the given triangle *ABC*, show that the altitudes of the triangles are concurrent.



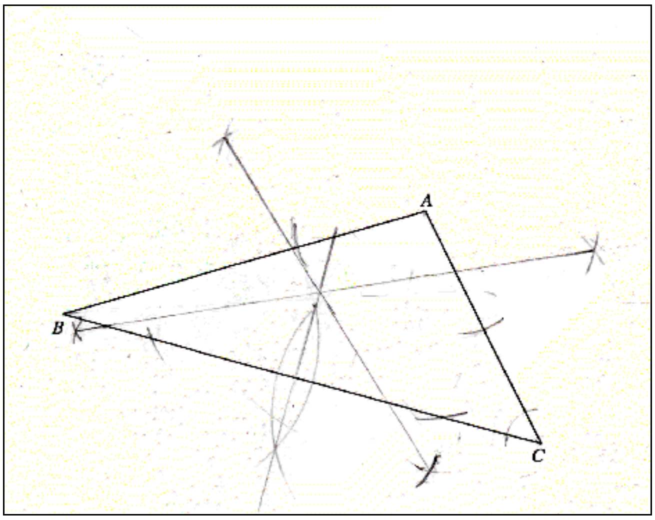
Better responses exhibited that candidates have good understanding of the construction of geometrical figures with the given measurements. They also constructed the required altitude correctly and were able to prove concurrency of the altitudes.

Example:

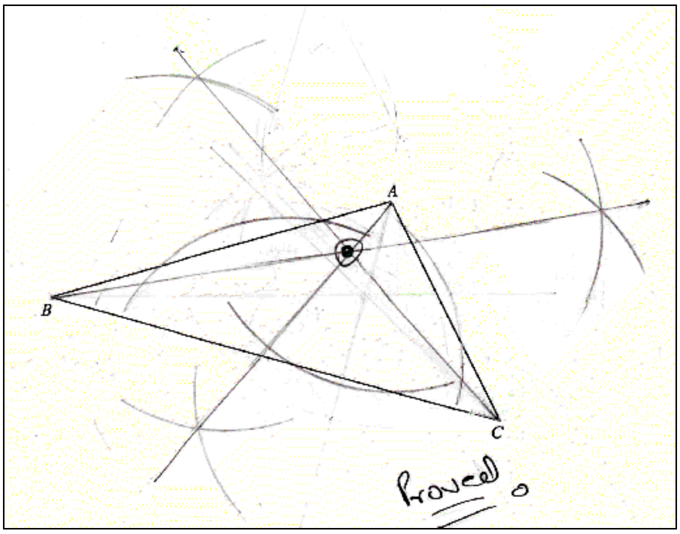


Weaker responses displayed that candidates were unable to draw the triangle the altitudes of the triangle as two altitudes required the extension of the sides and point of concurrency lies outside the given triangle. Other weaker responses displayed that candidates drew medians or angle bisectors instead of altitudes.

Example 1:



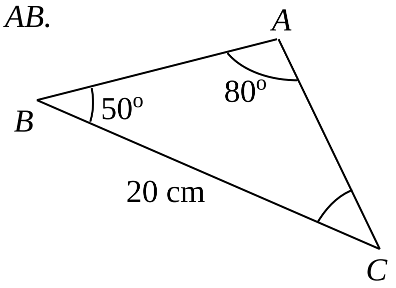
Example 2:



Question 9a:

In the given figure, *ABC* is a triangle and sum of all its sides are 30.56 cm.

Find the length of the side *AB*.



NOT TO SCALE

Better response reported that candidates understood the question and rightly applied the pertinent theorem to find angle *C* and length of side *AC*.

Example 1:

$\angle C + 80 + 50 = 180$	$\Rightarrow 20 + AB + AC = 30.56$
$\angle C + 130 = 180$	$\Rightarrow 20 + 2x = 30.56$
$\angle C = 180 - 130$	$\Rightarrow 2x = 30.56 - 20$
$\angle C = 50^\circ$	$\Rightarrow x = \frac{10.56}{2}$
$\angle C = \angle B$ so if base angles are	
equal so, opposite sides are	$\Rightarrow x = 5.28$ cm
also equal.	$\boxed{\text{Length of side } AB = 5.28 \text{ cm}}$

Example 2:

$20 + x + x = 30.56$
$20 + 2x = 30.56$
$2x = 30.56$
$x = \frac{30.56}{2}$
$x = 5.28$ cm

Weaker responses showed that the candidates were unable to understand the question and applied wrong approach or theorem to find the side of the triangle. The following examples cite different mistakes and misconceptions noted in the weaker responses.

Example 1:

$m\overline{AB} = ?$, $m\overline{BC} = 20cm$, $\angle B = 50°$, $\angle A = 80°$

$\overline{AB} + \overline{BC} = 30.56$

$AB + 20 = 30.56$

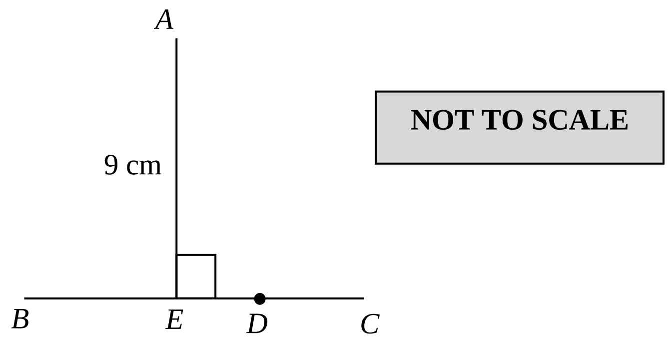
$AB = -20 + 30.56.$

$m\overline{AB} = -10.56$

Question 9b:

If lengths of any two sides of a triangle are 5 cm and 7 cm, then write any TWO possible lengths for the third side of the triangle.

ii. In the given diagram find any possible length of *AD*. Justify your answer

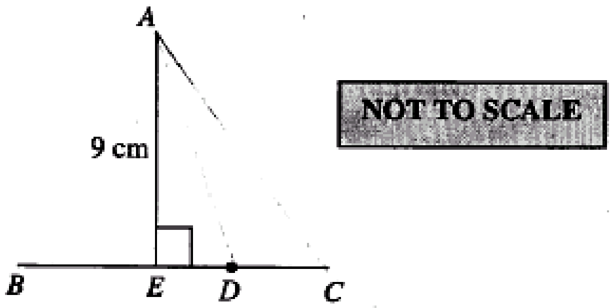


Better responses reported that candidates used the correct theorem and were able to establish the correct relation of *AE* and *AD.* They did not have any problem to the length of third side and possible values of *AD.*

Example 1:

5+7 = 12 , 5-7 = 2 , 1) 6 2) 8

ii. In the given diagram find any possible length of *AD*. Justify your answer (2 Marks)



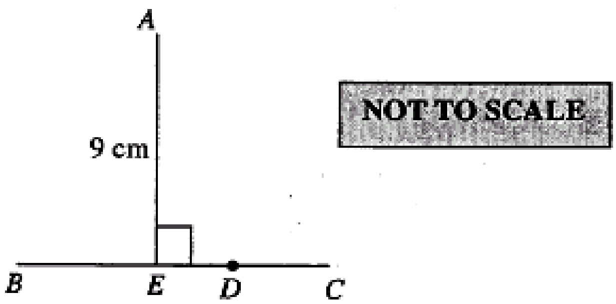
If AE = 9 , Then AD can be AD = 11

Because the line perpendicular to the other line is the shortest distance

Example 2:

1) 5cm, 7cm, 3cm Basically any value
2) 5cm, 7cm, 11cm in range of 3-11.

ii. In the given diagram find any possible length of *AD*. Justify your answer (2 Marks)



Possible length of $\overline{AD}$ can be 12 or any value more than 9cm as according to a theorem perpendicular distance is the shortest distance from a point to a line segment and $\overline{AE}$ is a perpendicular bisector.

Weaker responses showed different mistakes in comprehension and application of theorems and consequently, were failed to find the value of third side of the triangle and of *AD*.

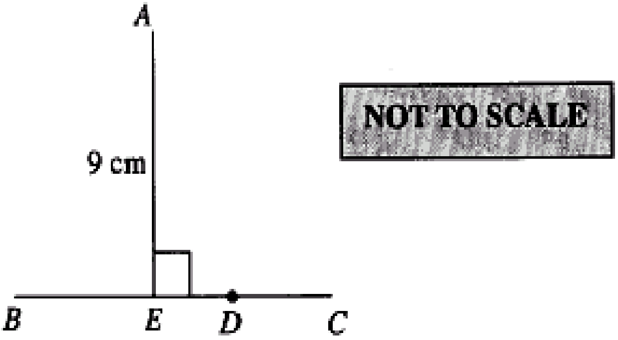
The following examples cite mistakes and misconceptions noted in the weaker responses.

Example 1:

~~[illegible]~~ third side can either be 12 or

$5+7 > mC$ 11.

ii. In the given diagram find any possible length of *AD*. Justify your answer (2 Marks)



According to the law AD will be greater than AE (9cm) because side opposite to larger angle will be larger. Hence 90° is the largest angle so side opposite to it (AD) will be largest.

Example 2:

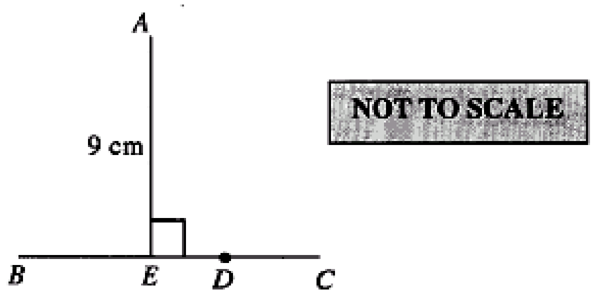
Sum of two sides of a triangle is greater than the third side

~~5+2~~ 5+7 = 12cm (Maximum) 7-5 = 2cm minimum

Possible length of third side of triangle can be 3 and 9

(in between the range of 2cm-12cm)

ii. In the given diagram find any possible length of *AD*. Justify your answer (2 Marks)



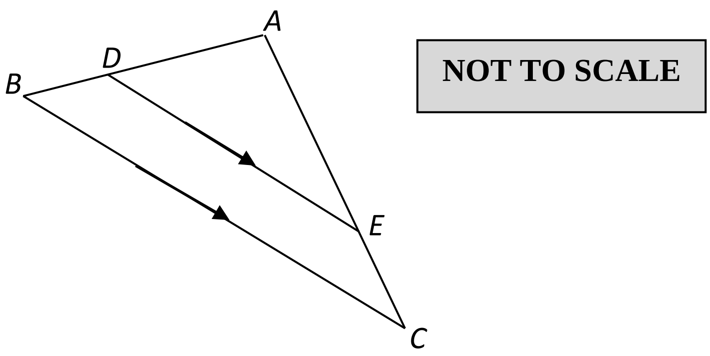
Possible length of $\overline{AD}$ can be 9cm since,

"Any point on the line segment of a right bisector is equidistance from its end point"

Question 9c:

In the given figure *ABC* is a triangle and *DE* is parallel to *BC*.

Find the length of *DB* if *AB* = 6 cm, *AC* = 8 cm and *EC* = 2 cm.



Better responses reported that candidates used the property of similar triangles or properties of parallel lines to develop the relations between sides of the triangles *ABC* and *ADE*.

$\frac{EC}{AE} = \frac{DB}{AD}$ and place the values properly to find the value of *DB*

Example 1:

$\Rightarrow \frac{BD}{AB} = \frac{EC}{AC}$.

$\Rightarrow \frac{x}{6} = \frac{2}{8}$

$\Rightarrow 8x = 6 \times 2$

$\Rightarrow 8x = 12$

$\Rightarrow x = \frac{\cancel{12}^{3}}{\cancel{8}_{2}}$

$\Rightarrow x = \frac{3}{2}$

$\Rightarrow x = 1.5cm$.

Example 2:

$\frac{AB}{DB} = \frac{AC}{EC}$ (The parallel lines DE and BC divides AB and AC ~~prop~~ in the ratio)

$\frac{6}{DB} = \frac{8}{2}$

$DB = \frac{\cancel{6}^{3} \times \cancel{2}^{1}}{\cancel{8}_{\cancel{4}_{2}}}$

$DB = \frac{3}{2} = 1.5$ Ans.

Example 3:

$\frac{AB}{BD} = \frac{AC}{EC}$ (similar triangles)

$\rightarrow \frac{6}{BD} = \frac{\cancel{8}^{4}}{\cancel{2}_{1}}$

$6 = BD \times 4 \Rightarrow \frac{\cancel{6}^{3}}{\cancel{4}_{2}} \rightarrow BD = 1.5cm$

Weaker responses showed different mistakes in writing or applying property of similar triangles or properties of parallel lines consequently, were failed to establish the correct relation between the sides of the triangle. They failed to find the measurement of side *DB*.

Example 1:

Given:-

ABC is a triangle.

DE is parallel to BC

AB = 6cm

AC = 8cm

EC = 2cm

find DB :-

AC = AB = 8cm

AD = 6cm

DB = 8cm − 6cm

$\boxed{DB = 2cm}$ answer:

Example 2:

length of DB = x

$\frac{AB}{DB} = \frac{AE}{EC}$

$\frac{6}{x} = \frac{6}{\cancel{4}\,2}$ => $6x = 12$ => $x = \frac{\cancel{12}\,2}{\cancel{6}\,1}$

AC = 8cm

AE = AC − EC

AE = 8 − 2 = 6

x = 2 Ans.