Aga Khan University Examination Board

Notes from E-Marking Centre on SSC I Mathematics Examination May 2015

Introduction

This document has been produced for the teachers and candidates of SSC Part I (Class IX) Mathematics. It contains comments on candidates' responses to the 2015 Secondary School Certificate (SSC-I) Examination, indicating the quality of the responses and highlighting their relative strengths and weaknesses.

E-Marking Notes

This includes overall comments on students' performance on every question and some specific examples of students' responses which support the mentioned comments. Please note that the descriptive comments represent an overall perception of the better and weaker responses as gathered from the e-marking session. However, the candidates' responses shared in this document represent some specific example(s) of the mentioned comments.

Teachers and candidates should be aware that examiners may ask questions that address the Student Learning Outcomes (SLOs) in a manner that require candidates to respond by integrating knowledge, understanding and application skills they have developed during the course of study. Candidates are advised to read and comprehend each question carefully before writing the response to fulfil the demand of the question.

Weaker responses revealed that candidates had problems with conversion of verbal phrases into mathematical operations to solve word problems, differentiating ungrouped and grouped data and properties of similar triangles. In general, the questions based on logarithms, laws of exponents, algebraic formulae and construction of triangles were not well attempted.

Detailed Comments:

Question 1:

This question offered a choice between part a and b. Majority of students chose part b which was also attempted better than part a.

Question 1a

Simplify $z = \dfrac{1}{-1+i} + 1 - 4i$ and express z in the form of $a + ib$.

Better responses exhibited thorough knowledge of conjugate of a complex number and application of basic operations on complex numbers to find least common multiple. The final answer was also expressed in the required form.

Example:

$$z = \frac{1}{-1+i} + 1 - 4i$$

$$z = \frac{1}{-1+i} \times \frac{-1-i}{-1-i} + 1 - 4i$$

$$z = \frac{-1-i}{(-1)^2 - (i)^2} + 1 - 4i$$

$$z = \frac{-1-i}{1-(-1)} + 1 - 4i$$

$$z = \left(\frac{-1-i}{2}\right) + 1 - 4i$$

$$z = \frac{-1-i+2-8i}{2}$$

$$z = \frac{1-9i}{2}$$

$$z = \frac{1}{2} - \frac{9i}{2}$$

$$a = \frac{1}{2}$$

$$ib = -\frac{9i}{2}$$

Weaker responses did not rationalise the denominator and had little or no understanding of conjugate of a complex number. Candidates had difficulty finding the least common multiple showing some candidates did not have a strong hold on basic operations on complex numbers.

Example:

$$z = \frac{1}{-1+i} + 1 - 4i$$

$$z = \frac{1}{i+i} + 1 - 4i$$

$$z = \frac{1}{i^2} + 1 - 4i$$

$$z = \frac{1}{1} + 1 - 4i \quad i^2$$

$$z = \frac{\cancel{1}}{\cancel{1}} + 1 - 4i$$

$$z = +1 - 4i$$

Ans!

Question 1b

Simplify $\sqrt{\dfrac{128m^{-3}n^{3}}{2m^{-7}n^{9}}}$, giving your answer in **positive** exponents.

Better responses correctly applied the laws of exponents and expressed the answer in the given form. Most of the responses simplified the term within the bracket first and later expressed square root as a power. However, a few responses showed that the candidates were able to do it the other way around as well.

Example:

$$\sqrt{\frac{128m^{-3}n^{3}}{2m^{-7}n^{9}}}$$

$$\Rightarrow \sqrt{\frac{64m^{-3}n^{3}}{m^{-7}n^{9}}}$$

$$\Rightarrow \sqrt{\frac{64}{m^{-7+3}n^{9-3}}}$$

$$\Rightarrow \sqrt{\frac{64}{m^{-4}n^{6}}}$$

$$\Rightarrow \left(\frac{2^{6}m^{4}}{n^{6}}\right)^{\frac{1}{2}}$$

$$\Rightarrow \frac{2^{6\times\frac{1}{2}}m^{4\times\frac{1}{2}}}{n^{6\times\frac{1}{2}}}$$

$$\Rightarrow \frac{2^{3}m^{2}}{n^{3}}$$

$$\Rightarrow \frac{8m^{2}}{n^{3}}$$

Weaker responses failed to exhibit the law of exponents in division; there was a vague understanding of change of sign rule but it could not be applied correctly resulting in loss of marks. In some of the responses, the square root was converted to exponent but was applied to the numerator only. Minor errors in simplification were frequent.

Example:

$$\sqrt{\frac{\cancel{128}^{64} m^{-3} n^{\cancel{8}1}}{\cancel{2}_{1} m^{-7} n^{\cancel{8}3}}}$$

$$\sqrt{\frac{64 m^{-\cancel{3}1} \cancel{n}}{m^{-7} \cancel{x}^{\cancel{3}1}}}$$

$$\sqrt{\frac{64m}{m^{-7}}}$$

$$\cancel{\sqrt{64(m^{5})^{-7}}}$$

$$64m^{-7}$$

Question 2

This question offered a choice between part a and b. Candidates chose to attempt both parts equally. Both parts of this question were attempted well by most of the candidates.

Question 2a

Two sets A and B are defined as $A = \{1, 2\}$ and $B = \{0, 1, 2\}$.

i. Find the cartesian product $A \times B$.
ii. Is $A \times B$ a function? Write a statement to justify your answer.
iii. Write an **into** function from A to B.
iv. If R is a binary relation such that the domain of $R = \{1\}$ and the range of $R = \{1, 2\}$, write down the relation R in $A \times B$.

Better responses reflected that candidates were able to understand the flow of the question. In part (i) all the elements of the Cartesian product of A and B were listed (six in all; the first element belonging to set A and the second to set B) separated by a comma and all the ordered pairs were enclosed in curly brackets. In part (ii), the candidates were able to state the right reason as to why $A \times B$ is not a function. Part (iii) displayed a strong knowledge of the types of functions and candidates had no difficulty linking it to part (i). Part (iv) exhibited clear understanding of domain and range of a binary relation. A large number of candidates used mapping diagrams to find their answers and got full credit for it.

Example:

i. Find the cartesian product $A \times B$. (1 Mark)

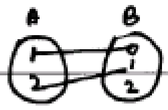
$A \times B = \{(1,0),(1,1),(1,2),(2,0),(2,1),(2,2)\}$

ii. Is $A \times B$ a function? Write a statement to justify your answer. (1 Mark)

No , it is not a function because in this domain is repeated but in function domain shouldnot be repeated so it is not afunction

iii. Write an **into** function from A to B. (1 Mark)

$\{(1,0)\ (2,1)\}$



Range $\subseteq$ B (into function)

iv. If R is a binary relation such that the domain of $R = \{1\}$ and the range of $R = \{1,2\}$, write down the relation R in $A \times B$. (1 Mark)

$R_1 = \{(1,1)\ (1,2)\}$

Weaker responses reflected weak understanding of functions in general. Part (i) exhibited incomplete Cartesian products and frequently 0 was incorrectly used as *x*-coordinate in the Cartesian product. Part (ii) reflected a lot of guess work showing candidates could not clearly differentiate between a function and a binary relation. A variety of responses came up in part (iii) which showed confusion between into, onto and one- one function. Some responses were not even a function. In part (iv), range was used in place of domain and domain in place of range.

Example:

i. Find the cartesian product $A \times B$. (1 Mark)

$A \times B$

$= \{1,2\} \times \{0,1,2\}$

$= \{0,1,2,0,2,4\}$

ii. Is $A \times B$ a function? Write a statement to justify your answer. (1 Mark)

Yes it is a function because Set A have no any extra element.

iii. Write an **into** function from A to B. (1 Mark)

$A \to B$, and $\bar{A}$ is the $\subseteq$ of B.

iv. If R is a binary relation such that the domain of $R = \{1\}$ and the range of $R = \{1,2\}$, write down the relation R in $A \times B$. (1 Mark)

Domain of $R = \{1,2\}$

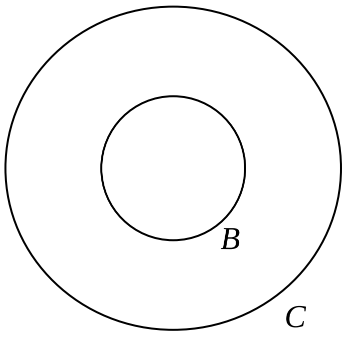
Range of $R = \{0,1,2\}$

Question 2b

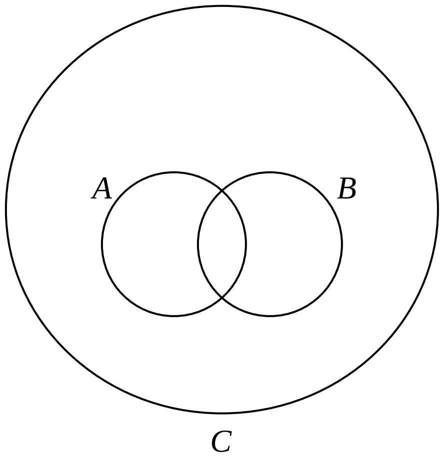
i. If $A=\{1,3,5,7\}$, $B=\{2,3,4,5\}$ and $C=\{1,2,3,4,5,6,7\}$, find $A\cap(B\cup C)$.

ii. Shade the following regions in the given Venn diagrams.

I. $B\cup C$



II. $A\cap(B\cup C)$



Better responses indicated the candidates not only had clear understanding of operations of union and intersection on sets but they were also able to exhibit them in the Venn diagrams. In part (i), only a few candidates used the distributive property of intersection over union; the majority found $(B \cup C)$ first which lead them to the correct answer. For part (ii), candidates used a variety of methods to find the shaded region. A large number of candidates used different patterns to shade A and $(B \cup C)$ which visually aided them to find the answer. A few candidates were able to grasp the fact that part (ii) was linked to part (i) and they found the shaded region by placing the elements of set A, B and C in the Venn diagram.

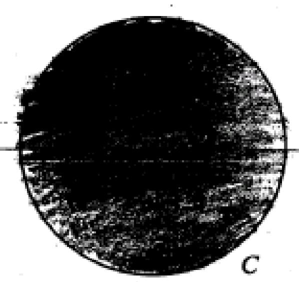
Example:

$(B \cup C) = \{2,3,4,5\} \cup \{1,2,3,4,5,6,7\} = \{1,2,3,4,5,6,7\}$

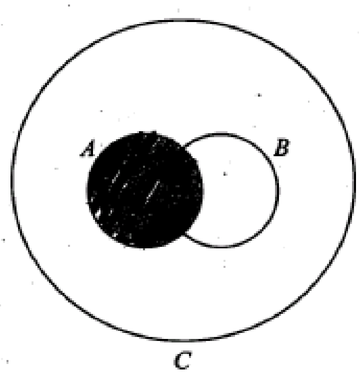
$A \cap (B \cup C) = \{1,3,5,7\} \cap \{1,2,3,4,5,6,7\}$

$= \underline{\{1,3,5,7\}}$ Ans.

I. $B \cup C$ **(1 Mark)**



II. $A \cap (B \cup C)$ **(1 Mark)**



Weaker responses for part (i) depicted a careless attitude toward an apparently easy question resulting in the candidates interchanging the signs of intersection and union or entirely ignoring one of these while copying the question. Some candidates failed to exhibit the operations of union and intersection or confused the two. Part (ii) reflected weak knowledge of how to represent union and intersection in Venn diagrams. Many candidates who shaded (I) correctly were unable to connect (II) with it. For (II) the most frequent incorrect response was a shaded set *A*'.

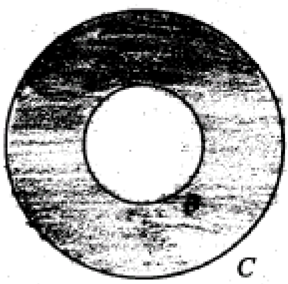
Example:

$A \cap (B \cup C) \cdot \{1,3,5,7\} (\{2,3,4,5\} \cup \{1,2,3,4,5,6,7\})$

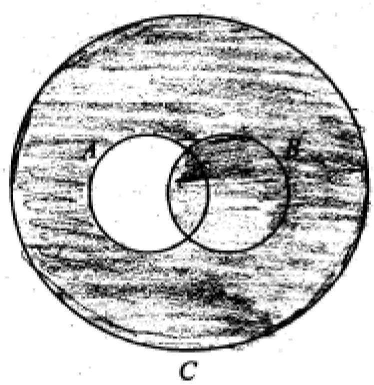
$\{1,3,5,7\} (\{2,3,4,5\})$

$\{1\times2+1\times3+1\times4+1\times5+3\times2+3\times3+3\times4+3\times5+5\times2+5\times3+5\times4+5\times5+7\times2+$
$7\times3+7\times4+7\times5\}) = \{14+42+70+98\} = \{224\}$

I. $B \cup C$ (1 Mark)



~~II. $A \cap (B \cup C)$ (1 Mark)~~



Question 3

Find the value of x if $\frac{1}{2}\log_4(3x-2)=\frac{1}{2}$.

Candidates exhibited average performance in this question.

Better responses exhibited correct use of the law $\log_a m^n = n\log_a m$ and correct conversion from logarithmic form to exponential form. Some candidates first reduced the given logarithmic equation to $\log_4(3x-2)=1$ and then converted it to exponential form. A few candidates expressed both sides of the given equation in the logarithmic form $\log_4\sqrt{(3x-2)}=\log_4 4^{\frac{1}{2}}$ which lead them to the equation $\sqrt{3x-2}=4^{\frac{1}{2}}$.

Example:

$4^{1/2} = (3x-2)^{1/2}$

$2^{\cancel{2}\times\frac{1}{\cancel{2}}} = (3x-2)^{1/2}$ = Squaring both the sides

$(2)^2 = \left(\sqrt{3x-2}\right)^2$

$4 = 3x-2$

$4+2 = 3x$

$6 = 3x$

$\frac{\cancel{6}\,2}{\cancel{3}} = x \qquad 2 = x.$

Weaker responses reflected that majority of the candidates completely ignored $\frac{1}{2}$ on the left side of the equation and converted the given equation to $\frac{1}{2}\left[(4)^{\frac{1}{2}}\right] = 3x - 2$ which lead them to the answer $x = 1$. Some candidates who applied the laws of logarithm could not remove the square root correctly and got confused in the last step.

Example:

$$\sqrt{4^{1/2}} = 3x/2$$
$$\sqrt{2^{2 \times 1/2}} = 3x/2$$
$$\sqrt{2} = 3x/2$$
$$\frac{2\sqrt{2}}{3} = x.$$

Question 4

This question offered a choice between part a and b. Candidates performed well in this question. Majority of the students attempted part b which was also attempted better by students as compared to part a.

Question 4a

If $x + \frac{1}{x} = a$, find the value of $x^3 + \frac{1}{x^3}$.

Better responses indicated the correct application of the formula $(a+b)^3 = a^3 + 3ab(a+b) + b^3$ to find $x^3 + \frac{1}{x^3}$. Candidates were able to understand that they have to substitute $x + \frac{1}{x} = a$ in the equation to get the answer.

Example:

$$\left(x + \frac{1}{x}\right)^3 = (a)^3$$
$$x^3 + \frac{1}{x^3} + 3(x)\left(\frac{1}{x}\right)\left(x + \frac{1}{x}\right) = a^3.$$
$$x^3 + \frac{1}{x^3} + 3(a) = a^3$$
$$x^3 + \frac{1}{x^3} = a^3 - 3a$$

Weaker responses reflected no knowledge of how to connect the formula $(a+b)^3 = a^3 + 3ab(a+b) + b^3$ with $x^3 + \frac{1}{x^3}$. Some candidates were not able to express their answer in terms of *a* since they did not understand that substitution is required to get the final answer. A few candidates used the wrong formula $a^3 + b^3 = (a+b)\left(a^2 - ab + b^2\right)$ to solve the question.

Example:

$$x^3 + \frac{1}{x^3}$$

$$a^3 + b^3 = (a+b) = (a^2 + ab + b^2)$$

$$a^3 + b^3 = (x^3 + 1/x^3) = ((x^3)^2 + (x)(1/x) + (1/x^3)^2)$$

$$a^3 + b^3 = (x^3 + 1/x^3) = [(x^3)^2 + 1 + (1/x^3)^2]$$

$$a^3 + b^3 = (x^3 + 1/x^3) = (x^3)^2 + 1 + (1\ x^3)^2$$

$$a^3 + b^3 = (x^3 + 1) = (x^3)^2 + 1 + 1$$

$$a^3 + b^3 = (x^3 + 1) = (x^3)^2 + 2$$

Question 4b If the square of the sum of three quantities *a, b, c* is 40 and the sum of their squares is 16, find the value of $(ab + bc + ca)$.

Better responses converted the given problem into correct mathematical equations $(a+b+c)^2 = 40$ and $a^2 + b^2 + c^2 = 16$ which were then correctly substituted in the formula $(a+b+c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ac)$ to get the correct answer.

Example:

$$(a+b+c)^2 = 40$$

$$a^2 + b^2 + c^2 + 2ab + 2bc + 2ca = 40$$

$$16 + 2(ab + bc + ca) = 40$$

$$2(ab + bc + ca) = 40 - 16$$

$$2(ab + bc + ca) = 24$$

$$ab + bc + ca = \frac{\cancel{24}^{12}}{\cancel{2}_1}$$

$$ab + bc + ca = 12$$

Answer.

Weaker responses converted the given question into wrong mathematical equations. The most frequent were

i. $(a+b+c)^2 = (40)^2$ and $a^2 + b^2 + c^2 = 16$ which lead candidates to the wrong equation $16 + 2(ab + bc + ac) = 1600$ and the wrong answer 792.

ii. $(a+b+c)^2 = (16)^2$ and $a^2 + b^2 + c^2 = 40$ which lead them to the equation $40 + 2(ab + bc + ac) = 256$ and the wrong answer 108.

Example:

Data

$a + b + c = 40$

$(a+b+c)^3 = 16$

$ab+bc+ca = ?$

Solve:.

$a + b+c - a^3+b^3+c^3 - 2ab+2bc+2ca = 90°$

$a+b+c$ 40 - ..

$(a+b+c)^3 = 16$

$= 40 - 16$

$= 20 - 90°$

$= 70°$

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Question 5

This question offered a choice between part a and b. Candidates performed well in this question. Majority of the students chose part a, but part b was attempted better by students as compared to part a.

Question 5a

Factorize $64 - z^6$.

Better responses exhibited correct application of algebraic formulae. Some candidates expressed the given equation as $(8)^2 - (z^3)^2$ while others expressed it as $(4)^3 - (z^2)^3$. Both ways candidates were able to find the factors. Surprisingly, some candidates tried to factorize the solution further using perfect square method. It led them nowhere but they still got full credit for their solution.

Example:

(a) $64 - z^6$

$= 64 - z^6$

$= 2^6 - z^6$

$= (2^3)^2 - (z^3)^2$

$= (2^3 - z^3)(2^3 + z^3)$

$= \{(2)^3 - (z)^3\}\{(2)^3 + (z)^3\}$

$= \{(2-z)((2)^2 + (2)(z) + (z)^2\}\{(2+z)((2)^2 - (2)(z) + (z)^2\}$

$= \{(2-z)(4 + 2z + z^2)\}\{(2+z)(4 - 2z + z^2)$

$= (2-z)(4+2z+z^2)(2+z)(4-2z+z^2)$

$= (2-z)(2+z)(4+2z+z^2)(4-2z+z^2)$ ans.

Weaker responses reflected weak application of expansion of algebraic formulae. The two cases are given below:

i. Candidates who expressed equation in the form $(8)^2 - (z^3)^2$ mostly got the correct factors $(2^3 - z^3)(2^3 + z^3)$ but made some error in the final factors. The most frequent incorrect answers were $(2-z)(2+z)(4+4z+z^2)(4-4z+z^2)$ and $(2-z)(2+z)(8+4z+z^3)(8-4z+z^3)$.

ii. Candidates who expressed equation in the form $(4)^3 - (z^2)^3$ were not able to solve it further by prefect square method and failed to express it as $(2-z)(2+z)((z^2+4)^2 - (2z)^2)$.

Example:

$$64 - z^6$$

$$(64)^2 \mp (64)(z^6) + (z^{\cancel{6}\,3})^{\cancel{2}\,1}$$

$$4096 - 64z^6 + z^3$$

$$4032z^6 + z^3$$

$4032z^9$ Answer.

Question 5b

Find the possible values of p, if $(px-p)$ is divided by $\left(x-\dfrac{4}{p}\right)$ and the remainder is p^2-p.

Better responses exhibited strong understanding of remainder theorem and its application to find equation in terms of p and factorize it further to find the answer.

Example:

$x - \dfrac{4}{p} = 0.$

$x = \dfrac{4}{p}.$

$px - p = p^2 - p.$

$p\left(\dfrac{4}{p}\right) - p = p^2 - p.$

$4 - p = p^2 - p.$

$4 \cancel{- p} \cancel{+ p} = p^2.$

$4 = p^2$

$\sqrt{p^2} = \sqrt{4}$ ∴ Apply squareroot on both sides.

$p = \pm 2.$

$p = \{+2, -2\}.$

Weaker responses reflected confusion between dividend and divisor which led candidates to solve $(px-p)=0 \Rightarrow px=p \Rightarrow x=1$ which is wrong. Some candidates could not understand that they had to apply remainder theorem.

Example:

$$\frac{px-p}{x-\frac{4}{p}} = p^2-p$$

$$px-p = p^2-\cancel{p}\left(x-\frac{4}{\cancel{p}}\right)$$

$$px-p = p^2(x-4)$$

$$px-p = p^2x-4p^2$$

$$4p^2-p = p^2x-px$$

$$4p = px$$

$$p = \frac{px}{4}$$

Ans

Question 6

This question offered a choice between part a and b. Candidates chose to attempt both parts equally, however, 6a was attempted better than 6b.

Question 6a

If t^2 is inversely proportional to *s*, when $s = 16$ and $t = 4$, find the value of *t* when $s = 4$.

Better responses exhibited thorough understanding of inversely proportional relationships. Candidates used the data given to make correct substitutions and found the right answer.

Example:

(4 Marks)

Data

$t^2 \propto \frac{1}{s}$

$t^2 = \frac{k}{s}$

$t^2 \times s = k$

Solution

when value of $t = 4$ and $s = 16$

$(4)^2 \times 16 = k$

$16 \times 16 = k$

$256 = k$

value of t when $s = 4$

$t^2 = \frac{k}{s}$

$t^2 = \frac{256}{4}$

$t^2 = 64$

for t, taking square root on both sides

$\sqrt{t^2} = \sqrt{64}$

$t = \pm 8$

Result:

Hence the value of t is ± 8 when s is 4

Weaker responses reflected a confusion between inversely and directly proportional relationships which led majority of the candidates to construct the wrong equation $t^2 = ks$ and find the wrong value $k = 1$.

Example:

$(a+b)^2 = a^2 + 2ab + b^2$

$(a+b)^2 = (16)^2 + 2(16)(4) + (4)^2$

$(a+b)^2 = (4)^2 + 2(4)(2) + (2)^2$

$(a+b)^2 = 16 + 16 + 4$

$(a+b)^2 = (16+4)^2$

$(a+b)^2 = (20)^2 = (2)^2$

If t^2 is inversely proportional to s. when s = 4 and t = 2

Question 6b	
i.	Find the value of x if $5:2x = 3:2x-4$.
ii.	If $x:y = u:v$, prove that $3x+7y:3x-7y = 3u+7v:3u-7v$.

Better responses of part (i) equated product of extremes to product of mean to get a linear equation which was easily solved by the candidates. For part (ii), candidates successfully used the componendo-dividendo property on the given ratio to prove the given.

Example:

i. Find the value of x if $5:2x = 3:2x-4$. (2 Marks)

product of extremes = product of mean

$\cancel{2x}\ 5(2x-4) = 3\times 2x$

$10x - 20 = 6x$

$10x - 6x = 20 \qquad 4x = 20 \qquad x = \cancel{\frac{20}{4}}^{5} \qquad \boxed{x=5}$

ii. If $x:y = u:v$, prove that $3x+7y:3x-7y = 3u+7v:3u-7v$. (2 Marks)

$x:y = u:v \Rightarrow \frac{x}{y} = \frac{u}{v}$ multiplying both sides by $\frac{3}{7}$

$\frac{3x}{7y} = \frac{3u}{7v}$ using Compenendo-dividendo theorem

$\frac{3x+7y}{3x-7y} = \frac{3u+7v}{3u-7v} \Rightarrow 3x+7y : 3x-7y = 3u+7v : 3u-7v$ Hence proved

L.H.S=R.H.S

Weaker responses of part (i) indicated the candidates were not able to express the given question in the form $\frac{5}{2x} = \frac{3}{2x-4}$ the knowledge of which is expected to be carried forward from middle school. For part (ii) the responses reflected confusion among componendo, dividendo, and componendo-dividendo property. Some candidates were unable to multiply $\frac{x}{y} = \frac{u}{v}$ with $\frac{3}{7}$. A lot of guess-work was being done by candidates in this question.

Example:

$5x : 2x = 3 : 2x - 4 \qquad 20x = 12x$

$\frac{5x}{2x} = \frac{3(2x)}{4} \qquad \boxed{x = \frac{12}{20}}$ Ans

$5x(4) = 2x(6x)$

$x : y = u : v \qquad 3x + 7y : 3x - 7y = 3u + 7v : 3u - 7v$

$3x + 7y : 3x - 7y = 3u + 7v : 3u - 7v$

$10xy : 4xy = 10uv : 4uv$

$10xy : 4xy = 10uv : 4uv : 6xy : 6uv$

Question 7

What is the value of *x* and *k* in the following matrix equation?

$$\begin{bmatrix} 3 & x \\ 4 & 5 \end{bmatrix} + \begin{bmatrix} x & 3x \\ -9 & -5 \end{bmatrix} = k\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -2 & 0 \\ 5 & 1 \end{bmatrix}$$

This was not a well attempted question.

Better responses exhibited skills of addition subtraction and scalar multiplication in matrices. The candidates were further able to solve by equating corresponding elements of the matrix equation, which lead them to find the values of *x* and *k*.

Example:

$$\cancel{3x}\begin{bmatrix} 3 & x \\ 4 & 5 \end{bmatrix} + \begin{bmatrix} x & 3x \\ -9 & -5 \end{bmatrix} = k\begin{bmatrix} \cancel{k}1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -2 & 0 \\ 5 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 3+x & x+3x \\ 4-9 & 5-5 \end{bmatrix} = \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix} - \begin{bmatrix} -2 & 0 \\ 5 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 3+x & 4x \\ \cancel{4}-5 & 0 \end{bmatrix} = \begin{bmatrix} k+2 & 0-0 \\ 0-5 & k-1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 3+x & 4x \\ -5 & 0 \end{bmatrix} = \begin{bmatrix} k+2 & 0 \\ -5 & k-1 \end{bmatrix}$$

Taking corresponding sides equal.

$4x = 0$

$\Rightarrow x = \frac{0}{4}$

$\Rightarrow x = 0$

$k - 1 = 0$

$\Rightarrow k = 1.$

The value of $x = 0$

and $k = 1.$

Weaker responses indicated that candidates subtracted the matrices on the right hand side before multiplying the first matrix with *k*. On the left hand side, candidates multiplied the two matrices instead of adding them. A few candidates could not equate the corresponding elements of equal matrices to form equations and left the solution in the middle.

Example:

$$x = \begin{bmatrix} 3 & x \\ 4 & 5 \end{bmatrix} + \begin{bmatrix} x & 3x \\ -9 & -5 \end{bmatrix} = k\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -2 & 6 \\ 5 & 1 \end{bmatrix}$$

The value of x $\begin{bmatrix} 3 & x \\ 4 & 5 \end{bmatrix} + \begin{bmatrix} x & 3x \\ -9 & -5 \end{bmatrix}$

and the k :

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -2 & 0 \\ 5 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 3 & x \\ 4 & 5 \end{bmatrix} + \begin{bmatrix} x & 3x \\ -9 & -5 \end{bmatrix} = k\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -2 & 0 \\ 5 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 3x \\ 9 \end{bmatrix} + \begin{bmatrix} 3x^2 \\ +14 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} - \begin{bmatrix} -2 \\ 4 \end{bmatrix}$$

$$\begin{bmatrix} 6x^3 \\ 28 \end{bmatrix} \begin{bmatrix} -1 \\ 3 \end{bmatrix}$$

Question 8

A small office has a staff of 20 employees. The distance (km) of the office from their homes is shown in the table given below. Complete the table and find the standard deviation for this data.

Distance (km)	**Number of Employees**	**Class Mark**		
$1-4$	2	2.5		
$5-8$	1	6.5		
$9-12$	6	10.5		
$13-16$	10	14.5		
$17-20$	1	18.5		
Total		–		

It was not a well attempted question.

Better responses exhibited the knowledge that the values of $f\,x$; $f\,x^2$ or $(x-\bar{x})^2$; $f\,(x-\bar{x})^2$ were to be found with the help of the table. These values were further correctly substituted in the formula of standard deviation for grouped data to find the answer.

Example:

Distance (km)	Number of Employees (f)	Class Mark (x)	fx	$f(x-\bar{x})^2$
1 – 4	2	2.5	5	$2 \times 88.36 = 176.72$
5 – 8	1	6.5	6.5	29.16
9 – 12	6	10.5	63	11.76
13 – 16	10	14.5	145	67.6
17 – 20	1	18.5	18.5	43.56
Total	$\Sigma f = 20$	–	$\Sigma fx = 238$	$\Sigma f(x-\bar{x})^2 = 328.8$

$$A.M = \bar{X} = \frac{\Sigma fx}{\Sigma f} = \frac{238}{20} = 11.9$$

$(x-\bar{x})$	$(x-\bar{x})^2$
$2.5 - 11.9 = -9.4$	$(-9.4)^2 = 88.36$
$6.5 - 11.9 = -5.4$	$(-5.4)^2 = 29.16$
$10.5 - 11.9 = -1.4$	$(-1.4)^2 = 1.96$
$14.5 - 11.9 = 2.6$	$(2.6)^2 = 6.76$
$18.5 - 11.9 = 6.6$	$(6.6)^2 = 43.56$

$$S.D = \sqrt{\frac{\Sigma f(x-\bar{x})^2}{\Sigma f}}$$

$$= \sqrt{\frac{328.8}{20}}$$

$$= \sqrt{16.44}$$

$$S.D = 4.05$$

$$\boxed{S.D = 4.05}$$

Weaker responses displayed that majority of the candidates either used the formula of standard deviation for ungrouped data or they were unable to use the given table to proceed further toward a solution. A lot of guess work was done in filling the table. Out of the candidates who used the formula of standard deviation for grouped data, there were many that made errors in the formula. The most frequent error was finding $(f\,x)^2$ instead of $f\,x^2$ and $\left[f\left(x-\bar{x}\right)\right]^2$ instead of $f\left(x-\bar{x}\right)^2$.

Example:

Distance (km)	Number of Employees f	Class Mark	class boundary	commulative frequency
1 – 4	2	2.5	0.5 – 4.5	2
5 – 8	1	6.5	4.5 – 8.5	2+1 = 3
9 – 12	6	10.5	8.5 – 12.5	3+6 = 9
13 – 16	10	14.5	12.5 – 16.5	9+10 = 19
17 – 20	1	18.5	16.5 – 20.5	19+1 = 20
Total	20	–		

Standard deviation $= \sqrt{\frac{\Sigma(x-\bar{x})^2}{n}}$

$\bar{x} = \frac{\Sigma fx}{\Sigma f} = \frac{4+3+54+190+20}{20} = 13.55$

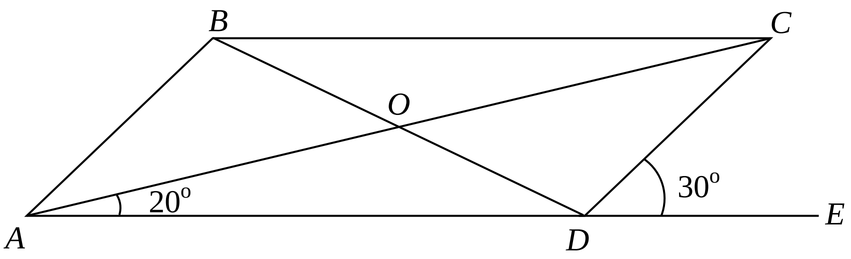
$\sqrt{\frac{\Sigma(x-\bar{x})^2}{n}}$

$\sqrt{\frac{(20-13.55)^2}{5}} = \frac{6.45}{5}$

$= 1.29.$

Question 9

A parallelogram $ABCD$ is given below. Find
i. $m\angle ABC$ (1 mark) ii. $m\angle OCD$ (1 mark) iii. $m\angle BAO$ (1 mark) iv. $m\angle BCA$ (1 mark)



NOT TO SCALE

This was not a well attempted question.

Better responses displayed knowledge that opposite angles are congruent in a parallelogram and applied this knowledge to find solutions for all parts.

Example:

i. $m\angle ABC$ (1 Mark)

$m\angle ABC + m\angle BAD = 180^\circ$

$m\angle ABC = 180^\circ - 30^\circ$

$= 150^\circ$

ii. $m\angle OCD$ (1 Mark)

$m\angle OCD = \cancel{150^\circ}\ 180^\circ - 150^\circ - 20^\circ$

$m\angle OCD = 10^\circ$

iii. $m\angle BAO$ (1 Mark)

$m\angle BAO = m\angle OCD$

$m\angle BAO = 10^\circ$

iv. $m\angle BCA$ (1 Mark)

$m\angle BCA = m\angle OAD$ ∴ corresponding angels

$m\angle BCA = 20^\circ$

Weaker responses reflected lack of knowledge of angles formed on a straight line which is included in middle school syllabus. The candidates could not work out how to use the given angles in the diagram and use properties of parallelogram to find the answer. Some candidates used a protractor to find the angles which was shocking since the diagram clearly states “NOT TO SCALE”.

Example:

i. $m\angle ABC$ (1 Mark)

$m\angle ABC = 180$ (∴ suplementary angle)

$m\angle 40^\circ + m\angle 60^\circ + m\angle 40^\circ = 180^\circ$

$m\angle 140^\circ = 180$ $m\angle ABC = 180 - 140^\circ = 40$ ∴ $m\angle ABC = 40^\circ$

ii. $m\angle OCD$ (1 Mark)

$m\angle 60^\circ + m\angle 40^\circ + m\angle 60^\circ = 180$ (∴ Suplementary angle) ∴ Vertically O.A

$m\angle 160^\circ = 180$

$m\angle 180^\circ - m\angle 160^\circ = 20^\circ$ ~~So~~ ∴ $m\angle OCD = 20^\circ$

iii. $m\angle BAO$ (1 Mark)

$m\angle 60 + m\angle 40 + m\angle 60^\circ = m\angle 180^\circ$ (∴ Supplementary angles) ∴ Vertically apposite angle

$m\angle 180 - m\angle 160^\circ = m\angle 20^\circ$

$m\angle BAO = 20^\circ$

iv. $m\angle BCA$ (1 Mark)

$m\angle 60^\circ + m\angle 40^\circ + m\angle 40^\circ = 180^\circ$ (∴ supplementary angles)

$m\angle 140^\circ = 180^\circ$

$m\angle 180^\circ - m\angle 140^\circ = m\angle 40^\circ$ ∴ $m\angle BCA = 40^\circ$

Question 10

This question offered a choice between part a and b. Candidates chose to attempt both parts equally**.** It was not attempted well by most of the candidates, however, part b was done slightly better than part a.

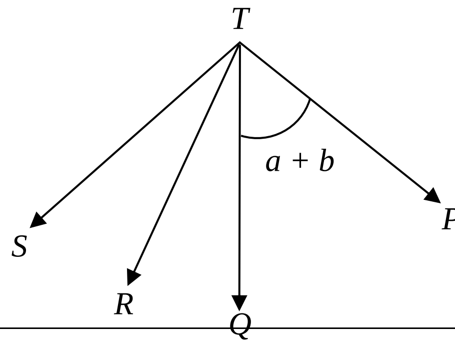
Question 10a

Given that TQ and TR are the angle bisectors of $\angle STP$ and $\angle STQ$ respectively, as shown in the diagram.

Find the following and write a statement to justify each of your answers.

i. $m\angle STR$

ii. $m\angle STP$



Better responses stated the angles correctly in terms of *a* and *b* justifying the answers with the correct reasoning that reflected complete and through concepts of angle bisectors.

Example:

i) $\angle STQ = a+b = \angle PTQ$; TQ is the angle bisector of $\angle STP$

which divides it into ~~to~~ 2 equal halves . ie $\angle STQ \cong \angle PTQ$.

=> As TR is the angle bisector of $\angle STQ$ it will divide it into half

ie $\frac{1}{2}(a+b) = \angle STR = \angle \cancel{Q}RTQ = \frac{a+b}{2}$.

$\angle STR = \frac{a+b}{2}$.

ii) $\angle STP$ = $\overline{TQ}$ divides $\angle STP$ into half ; $\angle PTQ = \frac{1}{2} \angle STP$.

=> $2\angle PTQ = \angle STP$.

=> $2(a+b) = \angle STP$. or $\angle STP = 2a+2b$.

Weaker responses reflected that most of the candidates were able to find the angles; however, the reasons stated were not correct. Some candidates assumed $m\angle STR$ and $m\angle RTQ$ are a and b respectively.

Example:

i. $m\angle STR = a°$ (because angle bisector divides an angle in two equal halves)

ii $m\angle STP = (a+b)+(a+b)$

$= a+b+a+b$

$= 2a+2b°$

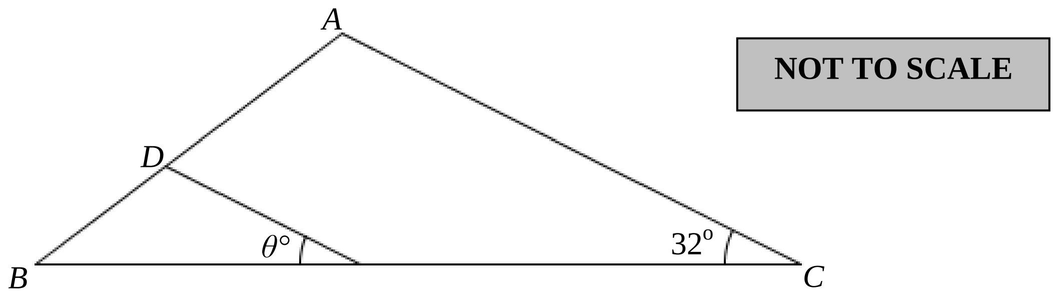
Question 10b

In triangle ABC, $m\overline{AB} = 5\,\text{cm}$, $m\overline{AD} = 3\,\text{cm}$, $m\overline{BC} = 10\,\text{cm}$, $m\overline{EC} = 6\,\text{cm}$, $m\overline{AC} = 6\,\text{cm}$ and $m\overline{DE} = \frac{12}{5}\,\text{cm}$.

i. Is triangle ABC similar to triangle DBE? Write a statement to justify your answer.

ii. Is $\overline{AC}$ parallel to $\overline{DE}$? Write a statement to justify your answer.

iii. Find the value of θ.



Better responses reflected strong concepts of similar triangles and their properties. Candidates were able to understand the question and justify it in every part.

Example:

i. Is triangle ABC similar to triangle DBE? Write a statement to justify your answer. (2 Marks)

$\frac{m\,AB}{DB} = \frac{m\,AC}{DE} = \frac{m\,BC}{BE}$

$\frac{5}{2} = \frac{6}{2.4} = \frac{10}{4}$

$2.5 = 2.5 = 2.5$

Yes, these triangles are similar to each other as their corresponding sides have similar ratio and their ~~angles~~ corresponding angles are also congruent.

ii. Is $\overline{AC}$ parallel to $\overline{DE}$? Write a statement to justify your answer. (1 Mark)

Yes, $\overline{AC}$ is parallel to $\overline{DE}$ because a line which passes joins two opposite sides of triangle divides the sides in equal ratio and is parallel to the third side of a triangle.

iii. Find the value of θ. (1 Mark)

$\theta^\circ = 32^\circ$ as $\Delta ABC \cong \Delta BDE$

$\therefore \angle C \cong \angle E$.

Weaker responses stated the correct answer in part (i) and (ii) but failed to justify the answers. Candidates wrote definition of similar triangles and straight lines in part (i) and (ii) respectively instead of reasons. In part (iii), candidates used guess-work which shows lack of knowledge regarding similar angles.

Example:

i. Is triangle *ABC* similar to triangle *DBE*? Write a statement to justify your answer. (2 Marks)

A. ABC is not similar to DBE because their angle & measurement of sides are not equal. ABC is the double of DBE & DBE is the half of ABC.

ii. Is $\overline{AC}$ parallel to $\overline{DE}$? Write a statement to justify your answer. (1 Mark)

$\overline{AC}$ is not parallel to $\overline{DE}$ because their measurement are not equal. AC = 6cm while $\overline{DE}$ = 2.4. so that's why it's not equal. $\overline{DE}$ is half of $\overline{AC}$.

iii. Find the value of θ. (1 Mark)

$\theta + 60 + 60 = 180$

$\theta = 180 - 120$

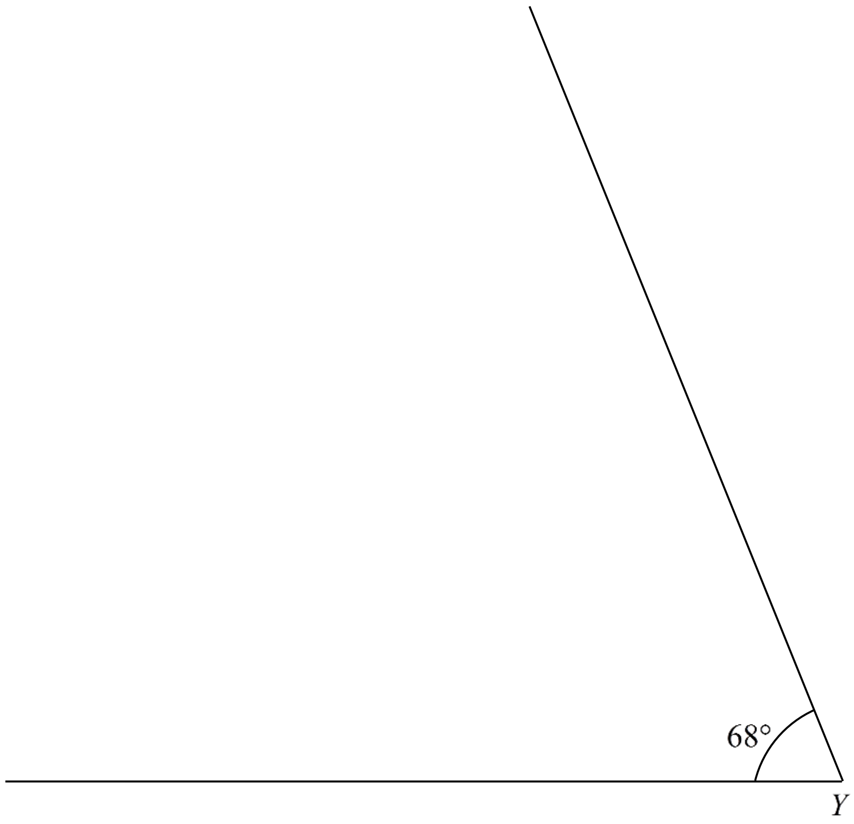
$\theta = 60$

Question 11

Given below is an incomplete scaled diagram of a triangle *XYZ* such that $m\,\overline{XY} = 8.8$ cm, $m\angle X = 56°$ and $m\angle Y = 68°$.

i. Complete the given diagram of the triangle. (2 marks)

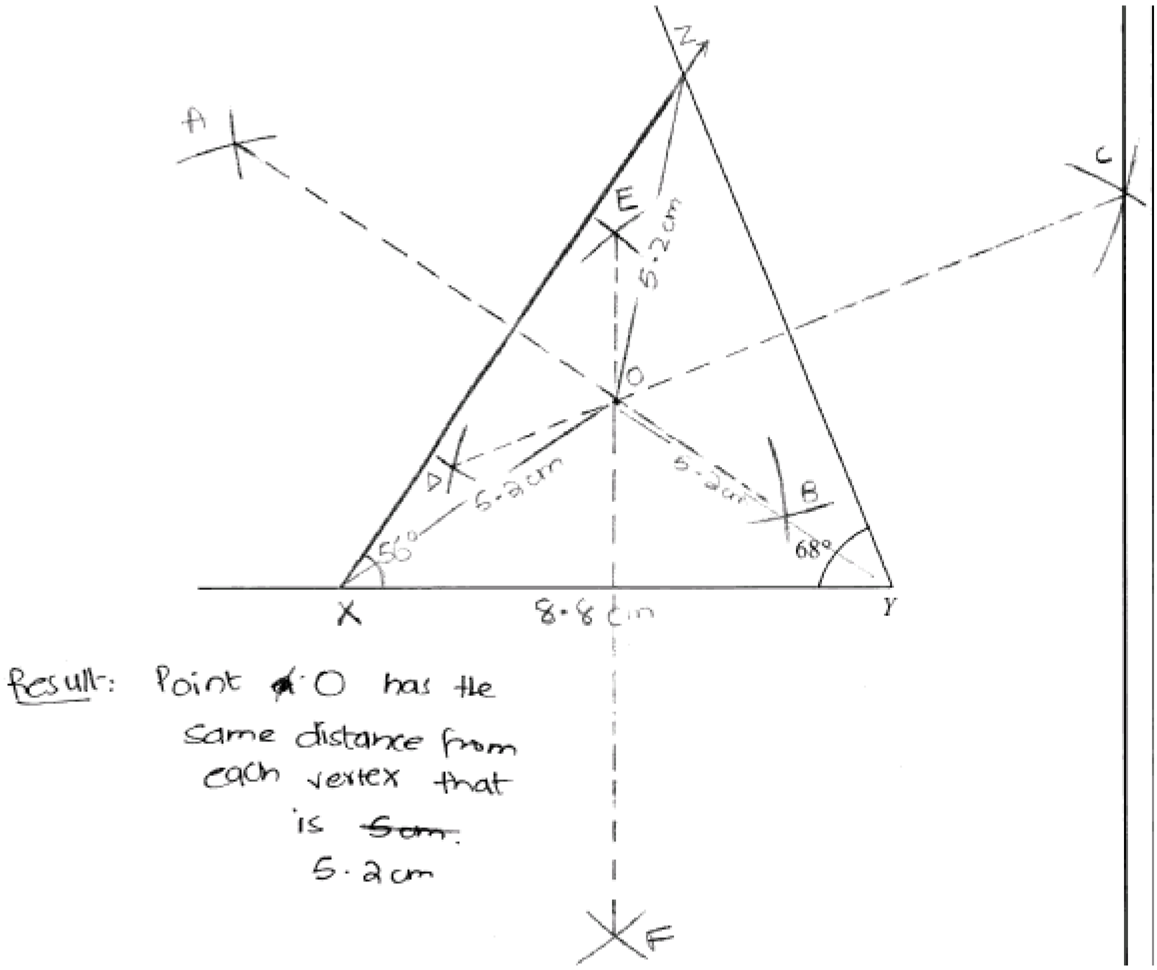
ii. Locate a point *O* such that *O* is at the same distance from each vertex of the triangle. Write down the distance of point *O* from any vertex.(2 marks)



Candidates exhibited average performance in this question.

Better responses demonstrated that the candidates were able to understand that they had to locate point *X* such that $\overline{XY} = 8.8$ cm. Geometrical instruments were used to complete the diagram accurately. Candidates also understood that *O* will lie at the point of intersection of perpendicular bisectors of at least two or all three sides of the triangle.

Example:



Weaker responses demonstrated two major mistakes.

i. The candidates assumed point X lies on the end points of lines given in the incomplete diagram. They did not realize that as per the question, $\overline{XY} = 8.8$ cm whereas the lines given in the figure were 11.1 cm.

ii. Candidates constructed angle bisectors, altitudes and medians instead of perpendicular bisectors which reflect the inability to find a point which is equidistant from two or three points.

Example: